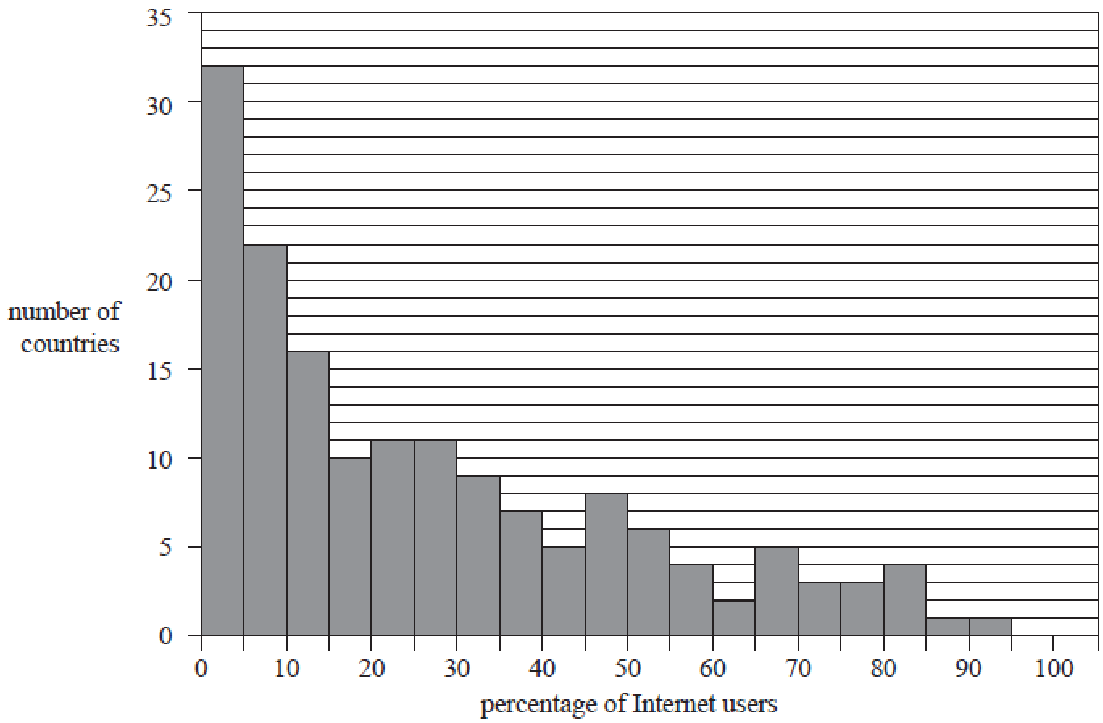
Core: Data Analysis

Use the following information to answer Questions 1, 2 and 3.

The histogram below displays the distribution of the percentage of Internet users in 160 countries in 2007.



Based on data obtained from: www.data.un.org

Question 1

The shape of the histogram is best described as

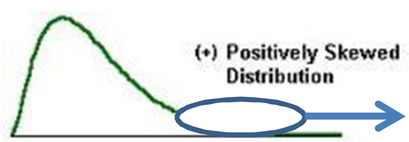
A. approximately symmetric.

B. bell shaped.

C. positively skewed.

D. negatively skewed.

E. bi-modal.



Tails off in the positive direction.

Section A: continu

Question 2

The number of countries in which less than 10% of people are Internet users is closest to

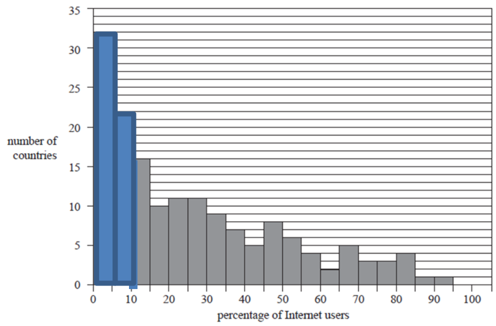
A. 10

B. 16

C. 22

D. 32

E. 54



Under 10%

22 +32 = 54 countries

80 countries are below 20.

Question 3

From the histogram, the median percentage of Internet users is closest to

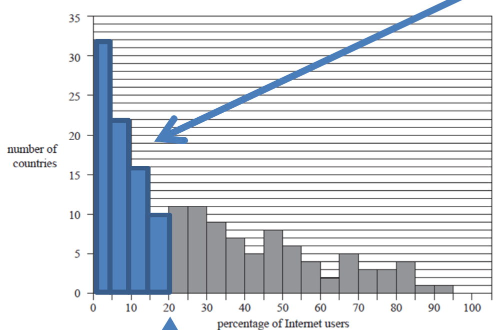
A. 10%

B. 15%

C. 20%

D. 30%

E. 40%



160 countries. So the median of the variable percentage of internet users will have ½ of 160 = 80 countries either side of the median value.

Add the frequencies until at least 80.

32 +22 + 16 + 10 = 80. This gives the median equal to 20%.

Median =20%

Region is a categorical variable with 3 levels (city, urban, rural).

Population density is a numerical variable.

Question 4

The variables

region (city, urban, rural)

population density (number of people per square kilometre)

A. are both categorical.

B. are both numerical.

C. are categorical and numerical respectively.

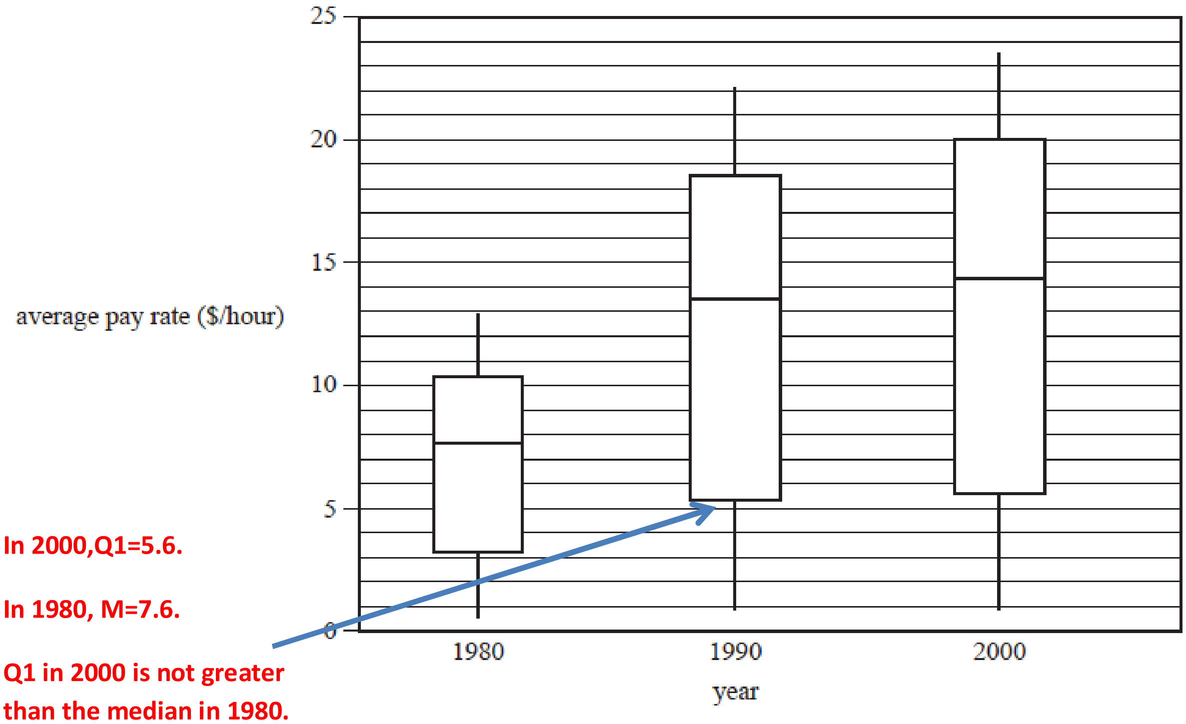
D. are numerical and categorical respectively.

E. are neither categorical nor numerical.

Section A: continued

Question 5

The boxplots below display the distribution of average pay rates, in dollars per hour, earned by workers in 35 countries for the years 1980, 1990 and 2000.



Based on data obtained from: www.data.un.org

In 2000,Q1=5.6.

In 1980, M=7.6.

Q1 in 2000 is not greater than the median in 1980.

Based on the information contained in the boxplots, which one of the following statements is **not** true?

A. In 1980, over 50% of the countries had an average pay rate less than $8.00 per hour. ✓

B. In 1990, over 75% of the countries had an average pay rate greater than $5.00 per hour. ✓

C. In 1990, the average pay rate in the top 50% of the countries was higher than the average pay rate for any of the countries in 1980. ✓

D. In 1990, over 50% of the countries had an average pay rate less than the median average pay rate in 2000. ✓

$\boxed{\textbf{E.}}$ In 2000, over 75% of the countries had an average pay rate greater than the median average pay rate in 1980. ✗

Section A: continued

Use the following information to answer Questions 6, 7 and 8.

When blood pressure is measured, both the systolic (or maximum) pressure and the diastolic (or minimum) pressure are recorded.

Table 1 displays the blood pressure readings, in mmHg, that result from fifteen successive measurements of the same person's blood pressure.

Table 1

	Blood pressure	
Reading number	**systolic**	**diastolic**
1	121	73
2	126	75
3	141	73
4	125	73
5	122	67
6	126	74
7	129	70
8	130	72
9	125	69
10	121	65
11	118	66
12	134	77
13	125	70
14	127	64
15	119	69

Question 6

Correct to one decimal place, the mean and standard deviation of this person's systolic blood pressure measurements are respectively

A. 124.9 and 4.4

B. 125.0 and 5.8

C. 125.0 and 6.0

D. 125.9 and 5.8

$\boxed{\textbf{E.}}$ 125.9 and 6.0

Use of calculator MENU 6 1 2 XLIST diastolic

$\bar{x}$	125.933333333	mean = 125.9
$"Sx := s_{n-1}x"$	5.95778802033	standard deviation = 6.0

Section A: continued

Question 7

Using systolic blood pressure (*systolic*) as the dependent variable, and diastolic blood pressure (*diastolic*) as the independent variable, a least squares regression line is fitted to the data in Table 1.

The equation of the least squares regression line is closest to

$\boxed{\textbf{A.}}$ $systolic = 70.3 + 0.790 \times diastolic$

B. $diastolic = 70.3 + 0.790 \times systolic$

C. $systolic = 29.3 + 0.330 \times diastolic$

D. $diastolic = 0.330 + 29.3 \times systolic$

E. $systolic = 0.790 + 70.3 \times diastolic$

Calculator: MENU 6 1 4 XLIST Diastolic YLIST Systolic

a	70.2861309138
b	0.78969538574
r^2	0.258188173669
r	0.508116004145

DV = a + b x IV

systolic =70.3 + 0.79 x *diastolic*

Question 8

From the fifteen blood pressure measurements for this person, it can be concluded that the percentage of the variation in systolic blood pressure that is explained by the variation in diastolic blood pressure is closest to

$\boxed{\textbf{A.}}$ 25.8%

B. 50.8%

C. 55.4%

D. 71.9%

E. 79.0%

Interpretation of r^2 in terms of explained variance.

$r^2 = 0.258 \times 100 = 25.8\%$

Use the following information to answer Questions 9 and 10.

The length of a type of ant is approximately normally distributed with a mean of 4.8 mm and a standard deviation of 1.2 mm.

Question 9

From this information it can be concluded that around 95% of the lengths of these ants should lie between

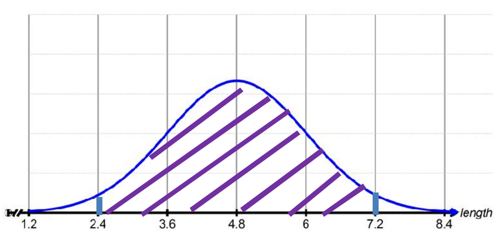
A. 2.4 mm and 6.0 mm

$\boxed{\textbf{B.}}$ 2.4 mm and 7.2 mm

C. 3.6 mm and 6.0 mm

D. 3.6 mm and 7.2 mm

E. 4.8 mm and 7.2 mm



95% of values lie within two standard deviations of the mean.

$$\bar{x} \pm 2s = 4.8 \pm 2 \times 1.2$$
$$4.8 - 2 \times 1.2 = 2.4$$
$$4.8 + 2 \times 1.2 = 7.3$$

Question 10

A standardised ant length of $z = -0.5$ corresponds to an actual ant length of

A. 2.4 mm

B. 3.6 mm

$\boxed{\textbf{C.}}$ 4.2 mm

D. 5.4 mm

E. 7.0 mm

$$Z = \frac{x - \bar{x}}{s}$$

$$-0.5 = \frac{x - 4.8}{1.2}$$

Then use solve

$$Solve(-0.5 = \frac{x - 4.8}{1.2}, x)$$

$$x = 4.2$$

Alternatively

A z value of -0.5 means that the value is 0.5 standard deviations below the mean. So

$$x = 4.8 - 0.5 \times 1.2$$
$$= 4.2$$

Section A: continued

Question 11

For a group of 15-year-old students who regularly played computer games, the correlation between the time spent playing computer games and fitness level was found to be $r = -0.56$.

Moderate negative association. So studentys who spent more time playing games tended to less fit.

On the basis of this information it can be concluded that

A. 56% of these students were not very fit.

B. these students would become fitter if they spent less time playing computer games.

C. these students would become fitter if they spent more time playing computer games.

Both imply causation

D. the students in the group who spent a short amount of time playing computer games tended to be fitter.

E. the students in the group who spent a large amount of time playing computer games tended to be fitter.

Question 12

The seasonal index for headache tablet sales in summer is 0.80.

To correct for seasonality, the headache tablet sales figures for summer should be

A. reduced by 80%

B. reduced by 25%

C. reduced by 20%

D. increased by 20%

E. increased by 25%

$$\begin{aligned} deseasoanlised\ value &= \frac{value}{SI} \\ &= \frac{value}{0.8} \\ &= \frac{1}{0.8} \times value \\ &= 1.25 \times value \end{aligned}$$

This represents a 0.25 x 100 = 25% increase in the value.

Question 13

The table below shows the number of broadband users in Australia for each of the years from 2004 to 2008.

Year	2004	2005	2006	2007	2008
Number	1 012 000	2 016 000	3 900 000	4 830 000	5 140 000

Based on data obtained from: www.data.worldbank.org

A two-point moving mean, with centring, is used to smooth the time series.

The smoothed value for the number of broadband users in Australia in 2006 is

A. 2 958 000

B. 3 379 600

C. 3 455 500

D. 3 661 500

E. 3 900 000

Need 3 values centred at 2006

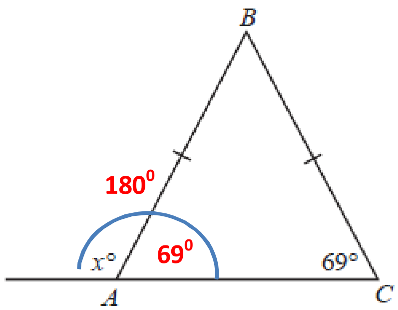
	2005	2006	2007
	2016000	3900000	4830000
MA(2)	2958000		4365000
Centred MA(2)		3661500	

END OF SECTION A

Module 2: Geometry and trigonometry

Question 1

In triangle ABC, $\angle BCA = 69°$ and $AB = BC$.



$x = 180 - 69$

$= 111$

The value of x is

A. 42

B. 55.5

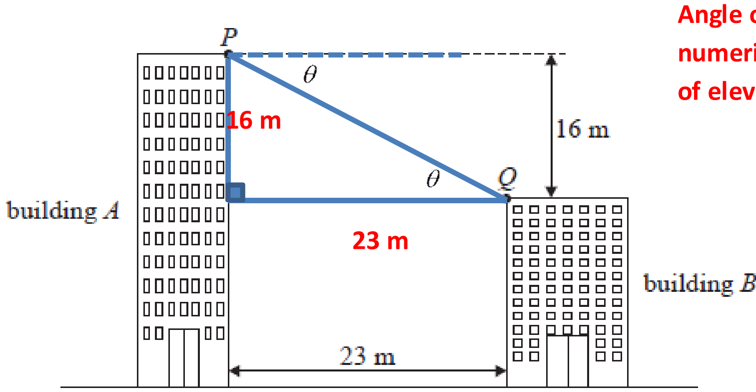
C. 84

$\boxed{\text{D.}}$ 111

E. 138

Question 2

Angle of depression numerically equals angle of elevation.



The point Q on building B is visible from the point P on building A, as shown in the diagram above.

Building A is 16 metres taller than building B.

The horizontal distance between point P and point Q is 23 metres.

The angle of depression of point Q from point P is closest to

$\boxed{\text{A.}}$ 35°

B. 41°

C. 44°

D. 46°

E. 55°

$\tan\theta = \frac{16}{23}$

$\theta = Tan^{-1}\left(\frac{16}{23}\right)$

$= 34.8$

Question 3

The radius of a circle is 6.5 centimetres.
A square has the same area as this circle.
The length of each side of the square, in centimetres, is closest to

A. 6.4
B. 10.2
C. 11.5
D. 23.0
E. 33.2

$A_{circle} = A_{square}$

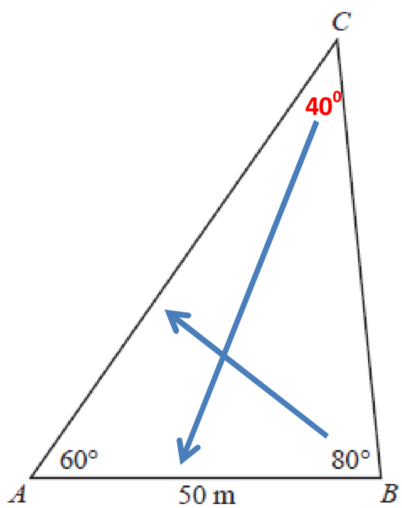
$\pi \times 6.5^2 = x \times x$

$Solve(\pi \times 6.5^2 = x \times x, x)$

$x = 11.5$

Question 4

In triangle ACB, $\angle CAB = 60°$ and $\angle ABC = 80°$
The length of side $AB = 50$ m.



Given two angles and a side so use the sine rule.

Firstly calculate angle C

C=180-60-80

$=40^0$

Applying the sine rule gives

$$\frac{AC}{\sin(80)} = \frac{50}{\sin(40)}$$

$$AC = \frac{50\sin(80)}{\sin(40)}$$

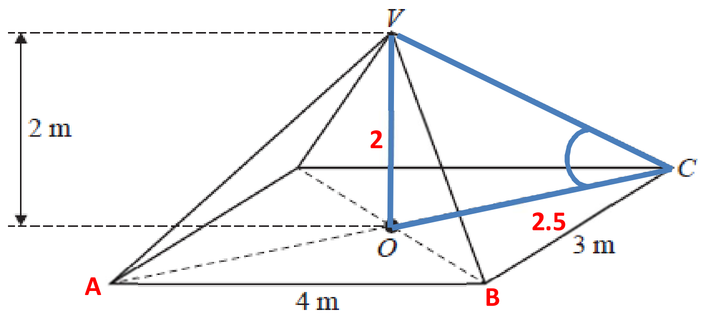
$$= 76.6$$

The length of side AC is closest to

A. 57 m
B. 67 m
C. 77 m
D. 81 m
E. 100 m

Question 5

A right pyramid, shown below, has a rectangular base with length 4 m and width 3 m.
The height of the pyramid is 2 m.



The angle *VCO* that the sloping edge *VC* makes with the base of the pyramid, to the nearest degree, is

A. 22°
B. 27°
C. 34°
D. 39°
E. 45°

ΔABC is a right angled triangle. Using Pythagoras to calculate AC gives

$$AC = \sqrt{4^2 + 3^2}$$
$$= 5$$

OC =1/2 x 5 = 2.5

$$\tan C = \frac{2}{2.5}$$
$$C = Tan^{-1}\left(\frac{2}{2.5}\right)$$
$$= 38.7$$

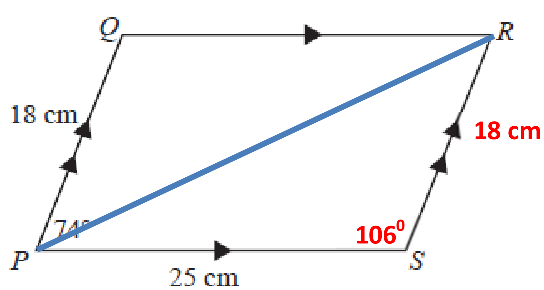
Question 6

In parallelogram *PQRS*, $\angle QPS = 74°$.
In this parallelogram, $PQ = 18$ cm and $PS = 25$ cm.

$\angle$P and $\angle$S are co-interior.

S=180-74

=106^0



SAS→S by Cosine rule.

$$PR^2 = 25^2 + 18^2 - 2 \times 25 \times 18 \times \cos(106)$$
$$PR = 34.6$$

The length of the longer diagonal of this parallelogram is closest to

A. 26.5 cm
B. 30.1 cm
C. 30.8 cm
D. 34.6 cm
E. 39.9 cm

Question 7

The structure of a roof frame is shown in the diagram below.

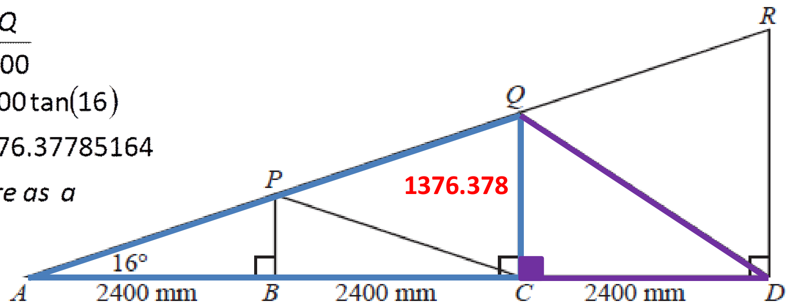
In this diagram, $AB = BC = CD = 2400$ mm and $\angle PAB = 16°$.

$$\tan(16) = \frac{CQ}{4800}$$

$$CQ = 4800\tan(16)$$

$$= 1376.37785164$$

Store as a



The length of QD, in mm, is closest to

A. 2741

$\boxed{\textbf{B.}}$ 2767

C. 2830

D. 3394

E. 5201

$$QD = \sqrt{a^2 + 2400^2}$$

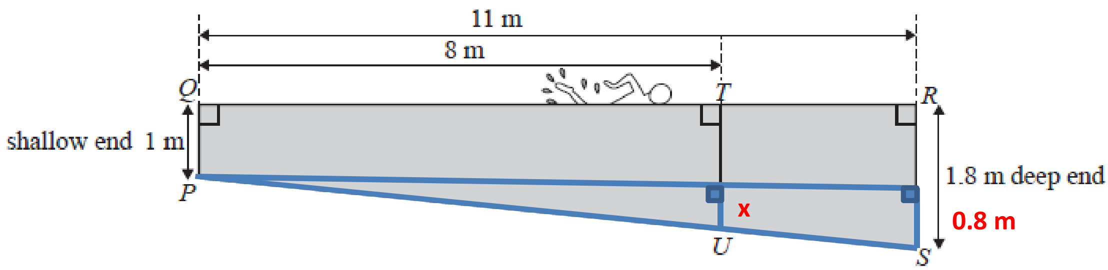
$$= 2766.7$$

Question 8

The diagram below shows a cross-section, $PQRS$, of a swimming pool.

The swimming pool is 11 metres long and the depth increases uniformly from 1 metre at the shallow end to 1.8 metres at the deep end.

Use similar triangles.



The depth of the water at a point 8 metres from the shallow end, represented by TU on the diagram, is closest to

A. 1.25 metres

B. 1.31 metres

C. 1.34 metres

$\boxed{\textbf{D.}}$ 1.58 metres

E. 1.62 metres

By similar triangles

$$\frac{x}{0.8} = \frac{8}{11}$$

$$x = \frac{8}{11} \times 0.8$$

$$= 0.58$$

Length TU = 1 + 0.58

= 1.58 metres

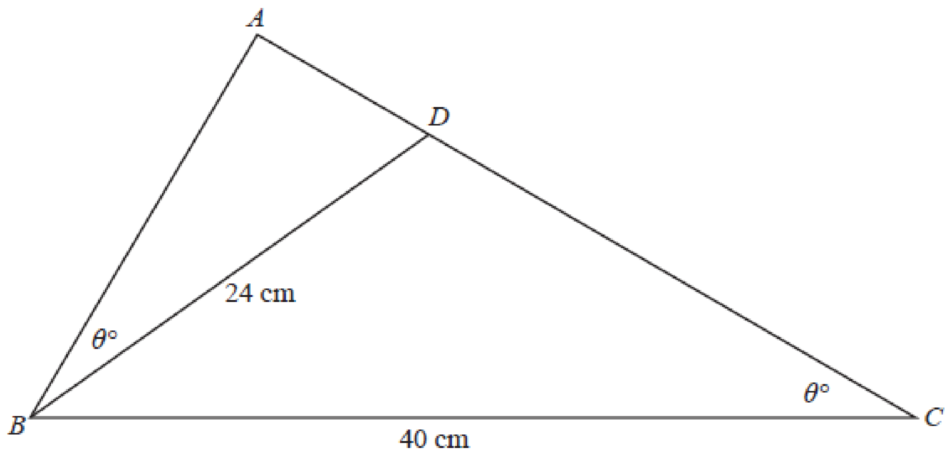
Question 9

In the diagram below, $\angle ABD = \angle ACB = \theta°$.

$BD = 24$ cm and $BC = 40$ cm.

The area of triangle ABD is 100 cm^2.

ΔABD is similar to ΔABC with side BD corresponding to side BC



The area of triangle ABC, in cm^2, is closest to

A. 167
B. 178
C. 267
D. 278
E. 378

	Small : Large
Lengths	24 : 40
	3 : 5
Areas	$3^2 : 5^2$
	9 : 25
	100 : A

$$\frac{A}{25} = \frac{100}{9}$$

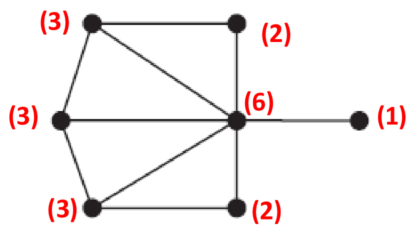
$$A = \frac{100}{9} \times 25$$

$$= 277.8$$

SECTION B - continued

Module 5: Networks and decision mathematics

Question 1

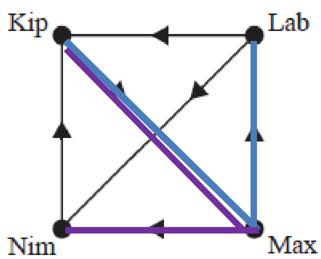


In the network shown, the number of vertices of even degree is

A. 2
B. 3
C. 4
D. 5
E. 6

Question 2

The graph below shows the one-step dominances between four farm dogs, Kip, Lab, Max and Nim.
In this graph, an arrow from Lab to Kip indicates that Lab has a one-step dominance over Kip.



2 step dominance implies 2 consecutive arrows.

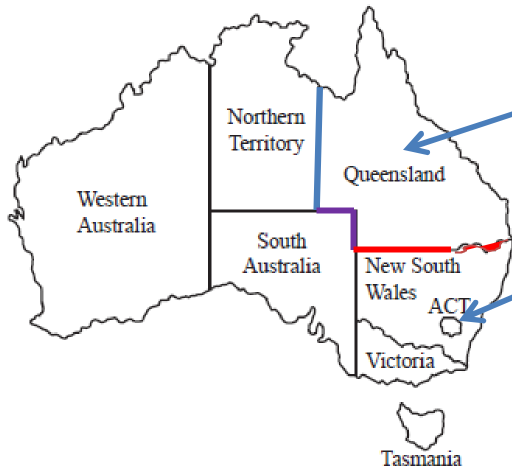
Kip has a two step dominance over Lab and Kim.

From this graph, it can be concluded that Kip has a two-step dominance over

A. Max only.
B. Nim only.
C. Lab and Nim only.
D. all of the other three dogs.
E. none of the other three dogs.

Use the following information to answer Questions 3 and 4.

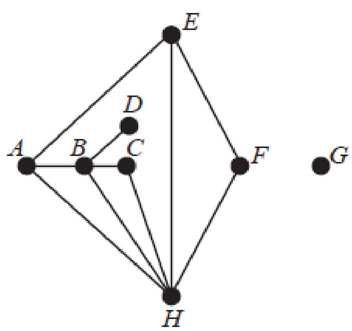
The map of Australia shows the six states, the Northern Territory and the Australian Capital Territory (ACT).



Queensland shares a border with the Northern Territory, South Australia and New South Wales.

ACT shares a border with NSW only. Its degree or order is 1.

In the network diagram below, each of the vertices A to H represents one of the states or territories shown on the map of Australia. The edges represent a border shared between two states or between a state and a territory.



ACT is shown by vertex D

Question 3

In the network diagram, the order of the vertex that represents the Australian Capital Territory (ACT) is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 4

In the network diagram, Queensland is represented by

A. vertex A.
B. vertex B.
C. vertex C.
D. vertex D.
E. vertex E.

Queensland has degree of 3 so either vertex A or E. Must be vertex A to represent Queensland as it shares borders with:

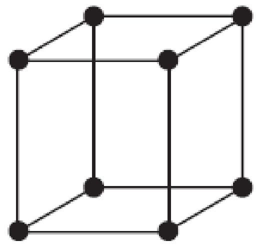
NSW vertex B (degree 4)

SA vertex H (degree 5)

NT vertex E (degree 3)

Question 5

A network is represented by the following graph.

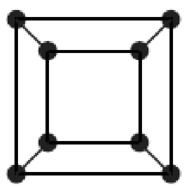


This graph has 8 vertices, each of degree 3, 12 edges and 6 regions.

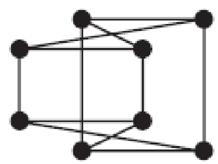
Require a graph that is not an isomorphic graph.

Which one of the following graphs could **not** be used to represent the same network?

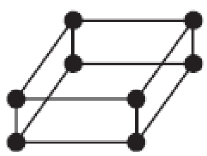
A.



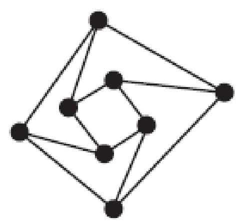
B.



C.

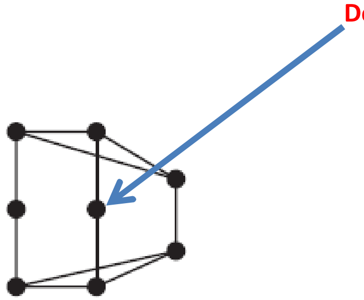


D.



Degree = 2

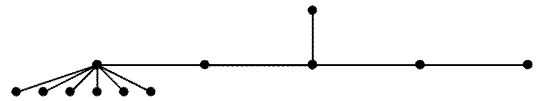
E.



Question 6

A store manager is directly in charge of five department managers.
Each department manager is directly in charge of six sales people in their department.

This staffing structure could be represented graphically by



A. a tree.
B. a circuit.
C. an Euler path.
D. a Hamiltonian path.
E. a complete graph.

Question 7

Andy, Brian and Caleb must complete three activities in total (*K*, *L* and *M*).

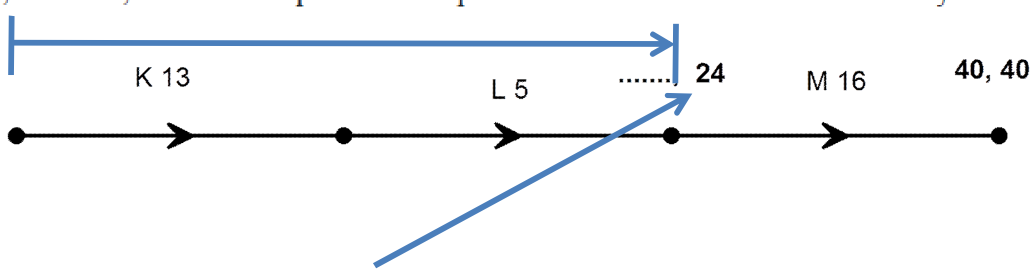
The table shows the person selected to complete each activity, the time it will take to complete the activity in minutes and the immediate predecessor for each activity.

Person	Activity	Duration	Immediate predecessor
Andy	*K*	13	–
Brian	*L*	5	*K*
Caleb	*M*	16	*L*

All three activities must be completed in a total of 40 minutes.
The instant that Andy starts his activity, Caleb gets a telephone call.

The maximum time, in minutes, that Caleb can speak on the telephone before he must start his allocated activity is

A. 5
B. 13
C. 18
D. 24
E. 34

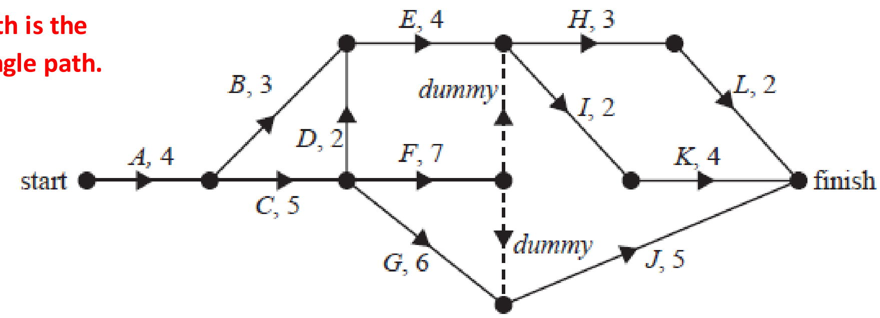


The latest finishing time for activity L is 24 minutes.
This represents the time that activity M must start.

Question 8

The diagram shows the tasks that must be completed in a project.
Also shown are the completion times, in minutes, for each task.

Critical path is the longest single path.



The critical path for this project includes activities

A. B and I.
B. C and H.
C. D and E.
$\boxed{\text{D.}}$ F and K.
E. G and J.

Paths:
ABEHL 4 + 3 + 4 + 3 + 2 = 16
ABEIK 4 + 3 + 4 + 2 + 4 = 17
ACDEHL 4 + 5 + 2 + 4 + 3 + 2 = 20
ACDEIK 4 + 5 + 2 + 4 + 2 + 4 = 21
ACFHL 4 + 5 + 7 + 3 + 2 = 21
ACFIK 4 + 5 + 7 + 2 + 4 = 22 Critical path
ACFJ 4 + 5 + 7 + 5 = 21
ACGJ 4 + 5 + 6 + 5 = 20

Question 9

An Euler path through a network commences at vertex P and ends at vertex Q.
Consider the following five statements about this Euler path and network.

An Euler line as starts and finishes at different vertices. Both P and Q are the only vertices of odd degree.

- In the network, there could be three vertices with degree equal to one. ✗
- The path could have passed through an isolated vertex. ✗
- The path could have included vertex Q more than once. ✓
- The sum of the degrees of vertices P and Q could equal seven. ✗
- The sum of the degrees of all vertices in the network could equal seven. ✗

The degree of P and Q both need to either odd.

How many of these statements are true?

A. 0
$\boxed{\text{B.}}$ 1
C. 2
D. 3
E. 4

Vertex P and Q could have degree of one, but all other vertices need to be of even degree

An isolated vertex has degree of zero. An Euler line must pass through all vertices.

Vertex Q needs to be of odd degree so could be 1, 3, 5 etc. So could include Q more than once.

Both P and Q need to be of odd degree. The sum of two odd numbers is always an even number.

The sum of degrees in any graph is always an even number as

$$\sum \text{degrees} = 2 \times E$$

SECTION B - continued

SECTION B – Module 6: Matrices

Question 1

The matrix below shows the airfares (in dollars) that are charged by Zeniff Airlines to fly between Adelaide (A), Melbourne (M) and Sydney (S).

$$\begin{array}{cc} & \textit{from} \\ & \begin{array}{ccc} A & M & S \end{array} \\ & \begin{bmatrix} 0 & 85 & 89 \\ 85 & 0 & 99 \\ 97 & \boxed{101} & 0 \end{bmatrix} \begin{array}{c} A \\ M \\ S \end{array} \quad \textit{to} \end{array}$$

Interpretation of the transition matrix

- **Column to row.**

The cost to fly from Melbourne to Sydney with Zeniff Airlines is

A. $85
B. $89
C. $97
D. $99
E. $101

Question 2

If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then $AB + 2C$ equals

A. $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Use of calculator or by hand as shown below

$$\begin{aligned} AB + 2C &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0\times1+1\times0 \\ 1\times1+0\times0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 3 \end{bmatrix} \end{aligned}$$

Question 3

Each of the following four matrix equations represents a system of simultaneous linear equations.

$$\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 1\times2-3\times0 = 2$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix} \checkmark$$

For a unique solution require the coefficient matrix to have a non zero determinant. So need to calculate the determinant of the four coefficient matrices.

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1\times2-1\times2 = 0$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix} \checkmark$$

$$\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1\times2-0\times0 = 2 \qquad \begin{vmatrix} 0 & 3 \\ 0 & 2 \end{vmatrix} = 0\times2-3\times0 = 0$$

$$\begin{bmatrix} 0 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

How many of these systems of simultaneous linear equations have a unique solution?

A. 0

B. 1

$\boxed{\textbf{C.}}$ 2

D. 3

E. 4

Question 4

Matrix A is a 3×4 matrix.

Matrix B is a 3×3 matrix.

Which one of the following matrix expressions is defined?

A. BA^2 ✗

$\boxed{\textbf{B.}}$ $BA - 2A$ ✓

C. $A + 2B$ ✗

D. $B^2 - AB$ ✗

E. A^{-1} ✗

Alternative A BA^2 **is not defined as** A^2 **which must be evaluated FIRST and is not defined-can only raise to a power a square matrix.**

Alternative B $BA-2A$ BA **is defined 3 x 3 3 x 4 to give a 3 x 4. So can then perform the subtraction as A is 3 x 4**

Alternative C $A+2B$ **is not defined as different orders (dimensions)**

Alternative D B^2-AB **is not defined as AB is not defined 3 x 4 3 x 4**

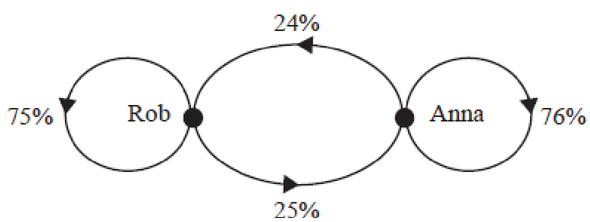
Alternative E A^{-1} **is not defined as can only take the inverse of a square matrix**

Use the following information to answer Questions 5 and 6.

Two politicians, Rob and Anna, are the only candidates for a forthcoming election. At the beginning of the election campaign, people were asked for whom they planned to vote. The numbers were as follows.

Candidate	Number of people who plan to vote for the candidate
Rob	5692
Anna	3450

During the election campaign, it is expected that people may change the candidate that they plan to vote for each week according to the following transition diagram.



Question 5

The total number of people who are expected to change the candidate that they plan to vote for one week after the election campaign begins is

A. 828
B. 1423
$\boxed{\textbf{C.}}$ 2251
D. 4269
E. 6891

After 1 week: 25% of Rob's supporters change to Anna and 24% of Anna's supporters change to Rob.

$$Total number = \frac{25}{100}\times 5692 + \frac{24}{100}\times 3450$$
$$= 2251$$

Question 6

The election campaign will run for ten weeks.

If people continue to follow this pattern of changing the candidate they plan to vote for, the expected winner after ten weeks will be

A. Rob by about 50 votes.
B. Rob by about 100 votes.
C. Rob by fewer than 10 votes.
D. Anna by about 100 votes.
$\boxed{\textbf{E.}}$ Anna by about 200 votes.

$$S_{10} = \begin{array}{c} \begin{array}{cc} R & A \end{array} \\ \begin{bmatrix} 0.75 & 0.24 \\ 0.25 & 0.76 \end{bmatrix}^{10} \end{array} \begin{array}{c} R \\ A \end{array} \times \begin{bmatrix} 5692 \\ 3450 \end{bmatrix} \begin{array}{c} R \\ A \end{array}$$
$$= \begin{bmatrix} 4479.2 \\ 4662.8 \end{bmatrix} \begin{array}{c} R \\ A \end{array}$$

After 10 weeks Rob receives 4479 votes and Anna 4663 votes. Anna wins by 4663-4479=184 (about 200 votes).

Question 7

Each night, a large group of mountain goats sleep at one of two locations, A or B.
On the first night, equal numbers of goats are observed to be sleeping at each location.
From night to night, goats change their sleeping locations according to a transition matrix T.
It is expected that, in the long term, more goats will sleep at location A than at location B.

This gives $S_1 = \begin{array}{cc} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array}$

Assuming the total number of goats remains constant, a transition matrix T that would predict this outcome is

Calculate T^{100} to give long term

$\boxed{\text{A.}}$

$$\begin{array}{ccc} & \textit{this night} & \\ & \begin{array}{cc} A & B \end{array} & \\ T = & \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night} \qquad T^{100} = \begin{bmatrix} 0.667 & 0.667 \\ 0.333 & 0.333 \end{bmatrix} \begin{array}{c} A \\ B \end{array}$$

B.

$$\begin{array}{ccc} & \textit{this night} & \\ & \begin{array}{cc} A & B \end{array} & \\ T = & \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night} \qquad T^{100} = \begin{bmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{bmatrix} \begin{array}{c} A \\ B \end{array}$$

C.

$$\begin{array}{ccc} & \textit{this night} & \\ & \begin{array}{cc} A & B \end{array} & \\ T = & \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night} \qquad T^{100} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \begin{array}{c} A \\ B \end{array}$$

D.

$$\begin{array}{ccc} & \textit{this night} & \\ & \begin{array}{cc} A & B \end{array} & \\ T = & \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night} \qquad T^{100} = \begin{bmatrix} 0.333 & 0.333 \\ 0.667 & 0.667 \end{bmatrix} \begin{array}{c} A \\ B \end{array}$$

E.

$$\begin{array}{ccc} & \textit{this night} & \\ & \begin{array}{cc} A & B \end{array} & \\ T = & \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night} \qquad T^{100} = \begin{bmatrix} 0.471 & 0.471 \\ 0.529 & 0.529 \end{bmatrix} \begin{array}{c} A \\ B \end{array}$$

SECTION B – Module 6: Matrices ï continued

Question 8

Consider the following matrix A.

$$A=\begin{bmatrix}3 & k\\ -4 & -3\end{bmatrix}$$

A is equal to its inverse A^{-1} for a particular value of k.

This value of k is

A. –4
B. –2
C. 0
D. 2
E. 4

Use calculator. Input matrix A. Then A^{-1} to give

$$A^{-1}=\begin{bmatrix}\frac{-0.75}{k-2.25} & \frac{-0.5625}{k-2.25}-0.25\\ \frac{1}{k-2.25} & \frac{0.75}{k-2.25}\end{bmatrix}$$

To simplify this use exact(A^{-1}) to give

$$A^{-1}=\begin{bmatrix}\frac{-3}{4k-9} & \frac{-9}{4(4k-9)}-\frac{1}{4}\\ \frac{4}{4k-9} & \frac{3}{4k-9}\end{bmatrix}$$

Since $A = A^{-1}$

$$\begin{bmatrix}3 & k\\ -4 & -3\end{bmatrix}=\begin{bmatrix}\frac{-3}{4k-9} & \frac{-9}{4(4k-9)}-\frac{1}{4}\\ \frac{4}{4k-9} & \frac{3}{4k-9}\end{bmatrix}$$

Now can equate elements in the same location.

So for instance $3=\frac{-3}{4k-9}$ which we can solve.

$Solve(3=\frac{-3}{4k-9},k)$ k= 2

B has dimensions 3 x 2 and AB will have dimensions 3 x 2

Question 9

Matrix A is a 3 × 3 matrix. Seven of the elements in matrix A are zero.
Matrix B contains six elements, none of which are zero.
Assuming the matrix product AB is defined, the minimum number of zero elements in the product matrix AB is

A. 0
B. 1
C. 2
D. 4
E. 6

Matrix A to minimise the number of zeroes in the answer will need to the smallest number of rows that have zeroes, that is one row. So A could possibly be

$$A=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 2\\ 0 & 0 & 0\end{bmatrix}$$ and B could be $$B=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

AB will have 2 elements that are zero.

$$AB=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 2\\ 0 & 0 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}5 & 6\\ 10 & 12\\ 0 & 0\end{bmatrix}$$

END OF MULTIPLE CHOICE QUESTION BOOK