



Victorian Certificate of Education 2012

FURTHER MATHEMATICS

Written examination 1

Friday 2 November 2012

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 40 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

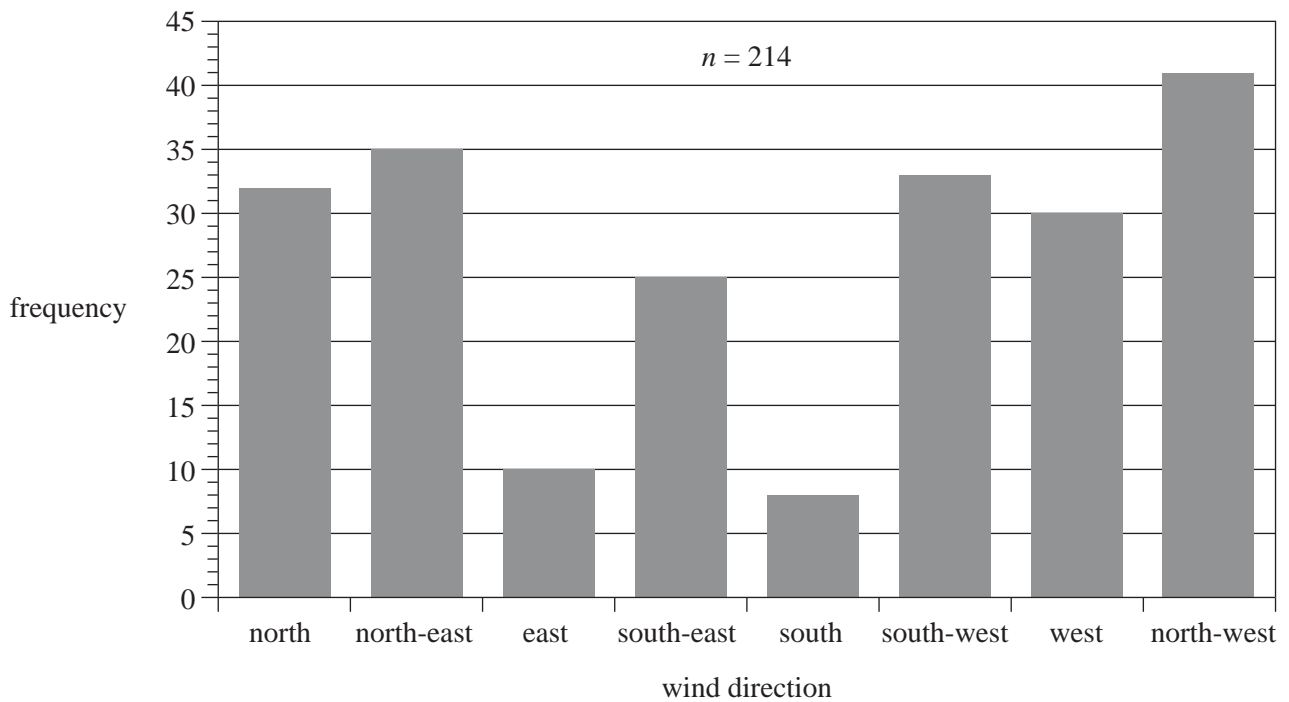
Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Core: Data analysis

Use the following information to answer Questions 1 and 2.

The following bar chart shows the distribution of wind directions recorded at a weather station at 9.00 am on each of 214 days in 2011.

**Question 1**

According to the bar chart, the most frequently observed wind direction was

- A. south-east.
- B. south.
- C. south-west.
- D. west.
- E. north-west.

Question 2

According to the bar chart, the percentage of the 214 days on which the wind direction was observed to be east or south-east is closest to

- A. 10%
- B. 16%
- C. 25%
- D. 33%
- E. 35%

Question 3

The total weight of nine oranges is 1.53 kg.

Using this information, the mean weight of an orange would be calculated to be closest to

- A. 115 g
- B. 138 g
- C. 153 g
- D. 162 g
- E. 170 g

Question 4

A class of students sat for a Biology test and a Legal Studies test. Each test had a possible maximum score of 100 marks. The table below shows the mean and standard deviation of the marks obtained in these tests.

	Subject	
	Biology	Legal Studies
Class mean	54	78
Class standard deviation	15	5

The class marks in each subject are approximately normally distributed.

Sashi obtained a mark of 81 in the Biology test.

The mark that Sashi would need to obtain on the Legal Studies test to achieve the same standard score for both Legal Studies and Biology is

- A. 81
- B. 82
- C. 83
- D. 87
- E. 95

Question 5

The temperature of a room is measured at hourly intervals throughout the day.

The most appropriate graph to show how the temperature changes from one hour to the next is a

- A. boxplot.
- B. stem plot.
- C. histogram.
- D. time series plot.
- E. two-way frequency table.

Question 6

The table below shows the percentage of households with and without a computer at home for the years 2007, 2009 and 2011.

	Year		
	2007	2009	2011
Households with a computer	66.4%	77.7%	84.5%
Households without a computer	33.6%	22.3%	15.5%

In the year 2009, a total of 5 170 000 households were surveyed.

The number of households **without** a computer at home in 2009 was closest to

- A. 801 000
- B. 1 153 000
- C. 1 737 000
- D. 3 433 000
- E. 4 017 000

Question 7

The table below shows the percentage of students in two age groups (15–19 years and 20–24 years) who regularly use the internet at one or more of three locations.

- at home
- at an educational institution
- at work

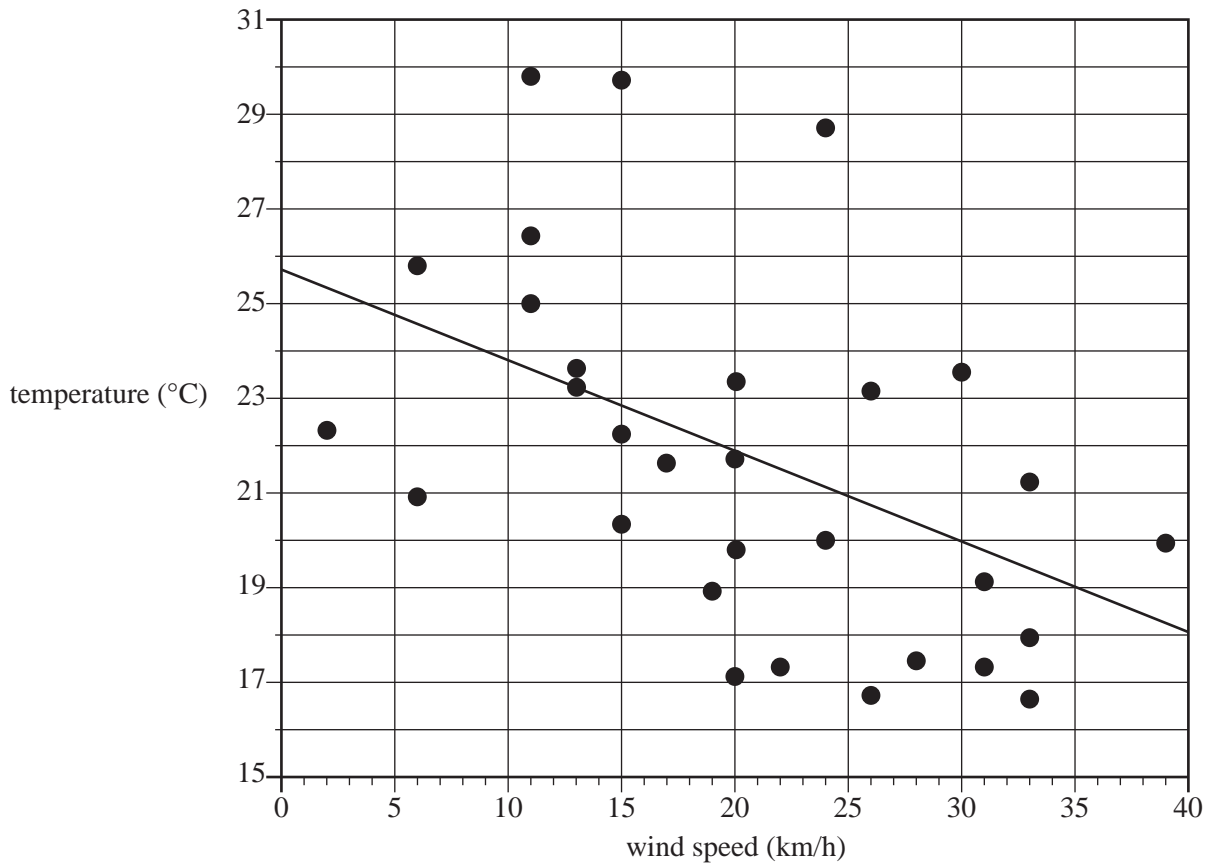
Location of internet use	Age group	
	15–19 years	20–24 years
at home	95%	95%
at an educational institution	85%	18%
at work	38%	74%

For the students surveyed, which one of the following statements, by itself, supports the contention that the location of internet use is associated with the age group of the internet user?

- A. 85% of students aged 15–19 years used the internet at an educational institution.
- B. 95% of students aged 15–19 years used the internet at home, but only 38% of 15–19 year olds used it at work.
- C. 95% of students aged 15–19 years used the internet at home and 18% of 20–24 year olds used the internet at an educational institution.
- D. The percentage of students who used the internet at an educational institution decreased from 85% for those aged 15–19 years to 18% for those aged 20–24 years.
- E. The percentage of students who used the internet at home was 95% for those aged 15–19 years and 95% for those aged 20–24 years.

Question 8

The maximum wind speed and maximum temperature were recorded each day for a month. The data is displayed in the scatterplot below and a least squares regression line has been fitted. The dependent variable is *temperature*. The independent variable is *wind speed*.

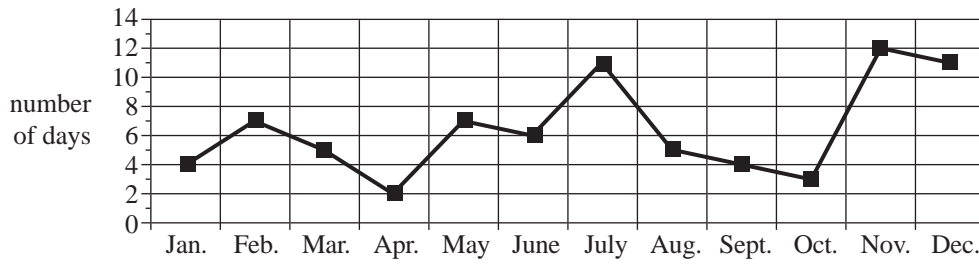


The equation of the least squares regression line is closest to

- A. $temperature = 25.7 - 0.191 \times wind\ speed$
- B. $wind\ speed = 25.7 - 0.191 \times temperature$
- C. $temperature = 0.191 + 25.7 \times wind\ speed$
- D. $wind\ speed = 25.7 + 0.191 \times temperature$
- E. $temperature = 25.7 + 0.191 \times wind\ speed$

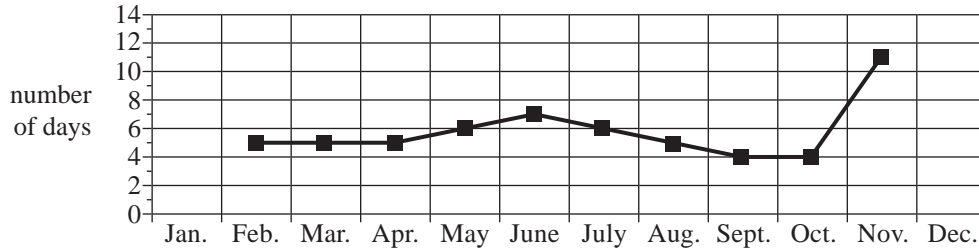
Question 9

The time series plot below shows the number of days that it rained in a town each month during 2011.

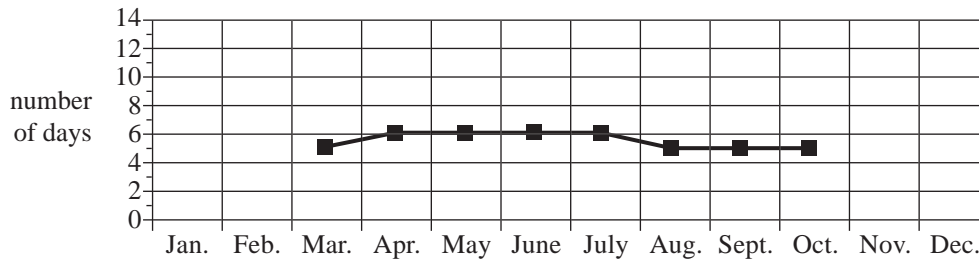


Using five-median smoothing, the smoothed time series plot will look most like

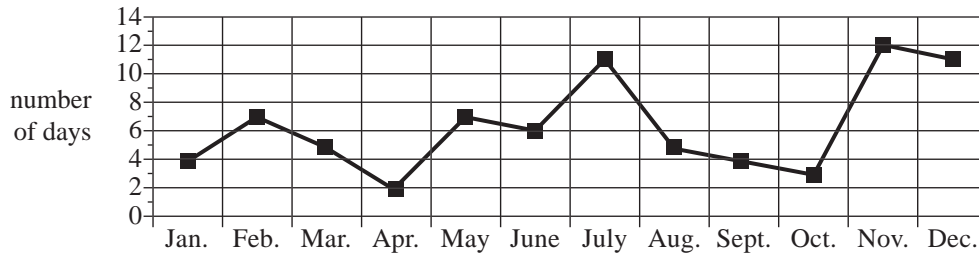
A.



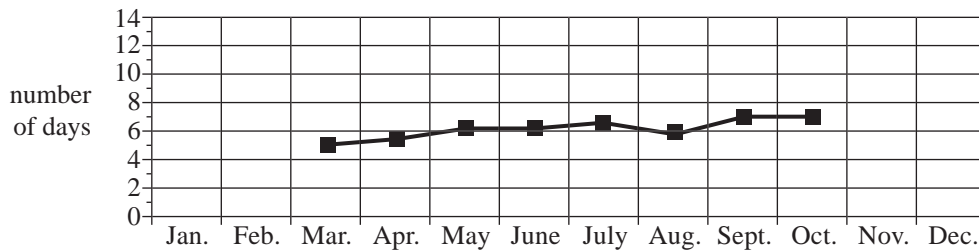
B.



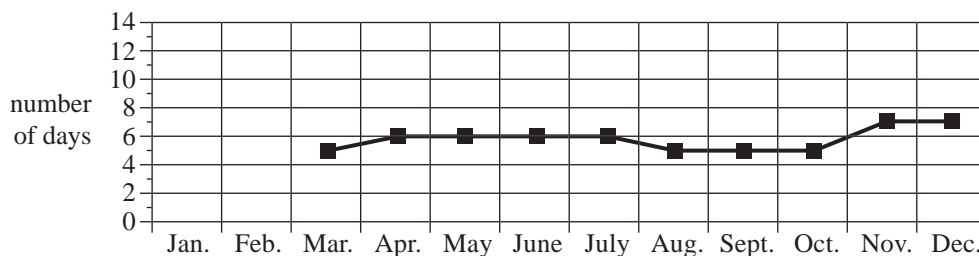
C.



D.



E.



Question 10

Which one of the following statistics is never negative?

- A. a median
- B. a residual
- C. a standardised score
- D. an interquartile range
- E. a correlation coefficient

Use the following information to answer Questions 11 and 12.

The table below shows the long-term average rainfall (in mm) for summer, autumn, winter and spring. Also shown are the seasonal indices for summer and autumn. The seasonal indices for winter and spring are missing.

	Season			
	Summer	Autumn	Winter	Spring
Long-term average rainfall (mm)	52.0	54.5	48.8	61.3
Seasonal index	0.96	1.01		

Question 11

The seasonal index for spring is closest to

- A. 0.90
- B. 1.03
- C. 1.13
- D. 1.15
- E. 1.17

Question 12

In 2011, the rainfall in autumn was 48.9 mm.

The deseasonalised rainfall (in mm) for autumn is closest to

- A. 48.4
- B. 48.9
- C. 49.4
- D. 50.9
- E. 54.0

Question 13

A trend line was fitted to a deseasonalised set of quarterly sales data for 2012.

The seasonal indices for quarters 1, 2 and 3 are given in the table below. The seasonal index for quarter 4 is not shown.

Quarter number	1	2	3	4
Seasonal index	1.2	0.7	0.8	

The equation of the trend line is

$$\text{deseasonalised sales} = 256\,000 + 15\,600 \times \text{quarter number}$$

Using this trend line, the **actual** sales for quarter 4 in 2012 are predicted to be closest to

- A. \$222 880
- B. \$244 923
- C. \$318 400
- D. \$382 080
- E. \$413 920

SECTION B**Instructions for Section B**

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Module	Page
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Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

1, 9, 10, 19, 29, . . .

The sixth term of the Fibonacci-related sequence shown above is

- A. 30
- B. 39
- C. 40
- D. 48
- E. 49

Question 2

A poultry farmer aims to increase the weight of a turkey by 10% each month.

The turkey's weight, T_n , in kilograms, after n months, would be modelled by the rule

- A. $T_{n+1} = T_n + 10$
- B. $T_{n+1} = 1.1T_n + 10$
- C. $T_{n+1} = 0.10T_n$
- D. $T_{n+1} = 10T_n$
- E. $T_{n+1} = 1.1T_n$

Use the following information to answer Questions 3 and 4.

As part of a savings plan, Stacey saved \$500 the first month and successively increased the amount that she saved each month by \$50. That is, in the second month she saved \$550, in the third month she saved \$600, and so on.

Question 3

The amount Stacey will save in the 20th month is

- A. \$1450
- B. \$1500
- C. \$1650
- D. \$1950
- E. \$3050

Question 4

The total amount Stacey will save in four years is

- A. \$13 400
- B. \$37 200
- C. \$58 800
- D. \$80 400
- E. \$81 600

Question 5

On the first day of a fundraising program, three boys had their heads shaved.

On the second day, each of those three boys shaved the heads of three other boys.

On the third day, each of the boys who was shaved on the second day shaved the heads of three other boys.

The head-shaving continued in this pattern for seven days.

The total number of boys who had their heads shaved in this fundraising activity was

- A. 2187
- B. 2188
- C. 3279
- D. 6558
- E. 6561

Question 6

The second and third terms of a geometric sequence are 100 and 160 respectively.

The sum of the first ten terms of this sequence is closest to

- A. 4300
- B. 6870
- C. 11 000
- D. 11 290
- E. 11 350

Question 7

A dragster is travelling at a speed of 100 km/h.

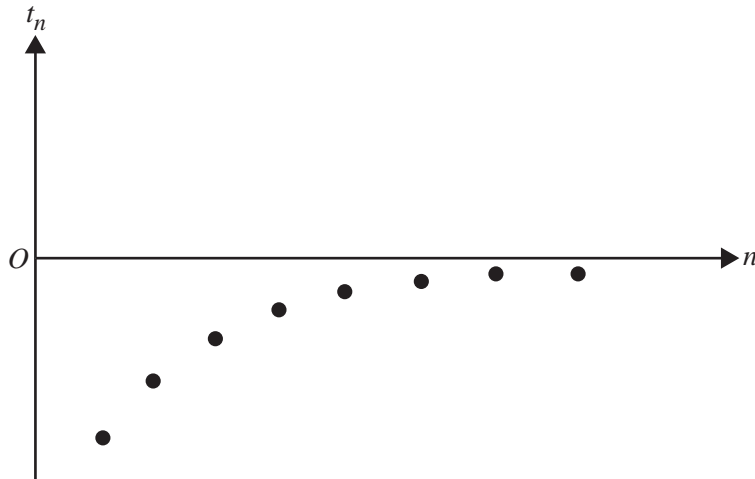
It increases its speed by

- 50 km/h in the 1st second
- 30 km/h in the 2nd second
- 18 km/h in the 3rd second

and so on in this pattern.

Correct to the nearest whole number, the greatest speed, in km/h, that the dragster will reach is

- A. 125
- B. 200
- C. 220
- D. 225
- E. 250

Question 8

The graph above shows consecutive terms of a sequence.

The sequence could be

- A. geometric with common ratio r , where $r < 0$
- B. geometric with common ratio r , where $0 < r < 1$
- C. geometric with common ratio r , where $r > 1$
- D. arithmetic with common difference d , where $d < 0$
- E. arithmetic with common difference d , where $d > 0$

Question 9

Three years after observations began, 12 300 birds were living in a wetland.

The number of birds living in the wetland changes from year to year according to the difference equation

$$t_{n+1} = 1.5t_n - 3000 \quad t_3 = 12\,300$$

where t_n is the number of birds observed in the wetland n years after observations began.

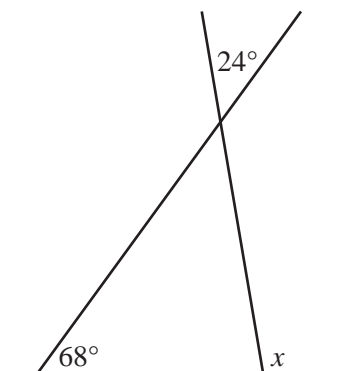
The number of birds living in the wetland one year after observations began was closest to

- A. 8800
- B. 9300
- C. 10200
- D. 12300
- E. 120175

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



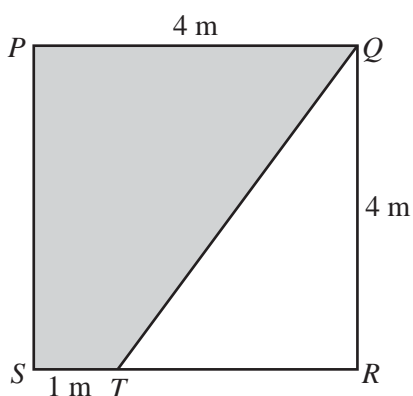
The size of the angle x is

- A. 68°
- B. 88°
- C. 92°
- D. 112°
- E. 116°

Question 2

$PQRS$ is a square of side length 4 m as shown in the diagram below.

The distance ST is 1 m.



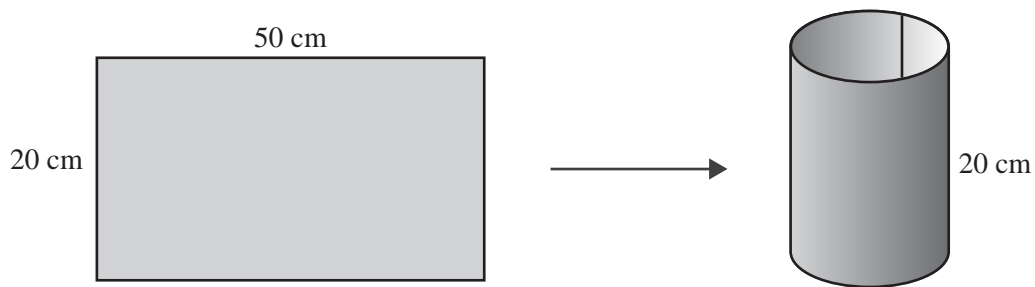
The shaded area $PQTS$ shown in the diagram, in m^2 , is closest to

- A. 6
- B. 8
- C. 9
- D. 10
- E. 12

Question 3

A rectangular sheet of cardboard has length 50 cm and width 20 cm.

This sheet of cardboard is made into an open-ended cylinder by joining the two shorter sides, with no overlap. This is shown in the diagram below.



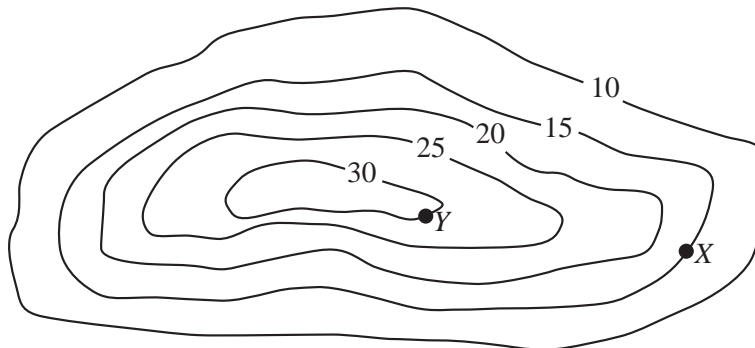
The radius of this cylinder, in cm, is closest to

- A. 6.4
- B. 8.0
- C. 15.6
- D. 15.9
- E. 17.8

Question 4

On the contour map below, the contour interval is 5 m.

The horizontal distance between the points X and Y is 1200 m.



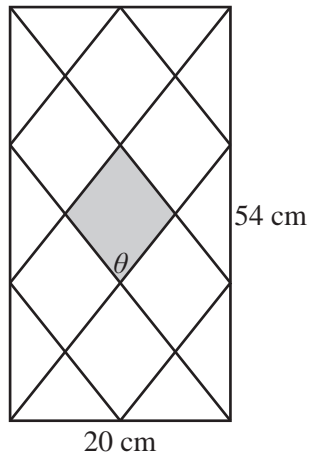
The average slope between X and Y is closest to

- A. 0.0042
- B. 0.0125
- C. 0.0167
- D. 0.0250
- E. 0.1250

Question 5

The rectangle shown below is 54 cm high and 20 cm wide.

The rhombuses drawn inside the rectangle are all the same size and shape.



The size of the angle θ , in the shaded rhombus, is closest to

- A. 34°
- B. 45°
- C. 56°
- D. 58°
- E. 67°

Question 6

A cylinder of radius R and height H has volume V .

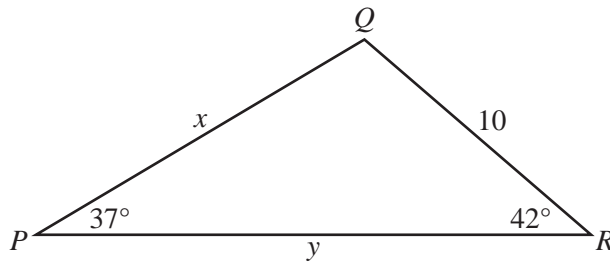
The volume of a cylinder with radius $3R$ and height $3H$ is

- A. $3V$
- B. $6V$
- C. $9V$
- D. $27V$
- E. $81V$

Question 7

PQR is a triangle with side lengths x , 10 and y , as shown below.

In this triangle, angle $RPQ = 37^\circ$ and angle $QRP = 42^\circ$.



Which one of the following expressions is correct for triangle PQR ?

- A. $x = \frac{10}{\sin 37^\circ}$
- B. $y = \frac{10}{\tan 37^\circ}$
- C. $x = 10 \times \frac{\sin 42^\circ}{\sin 37^\circ}$
- D. $y = 10 \times \frac{\sin 37^\circ}{\sin 101^\circ}$
- E. $10^2 = x^2 + y^2 - 2xy \cos 42^\circ$

Question 8

A triangular course for a yacht race has three stages.

Stage 1 is from the Start to Marker 1; a distance of 3.5 km on a bearing of 055° .

Stage 2 is from Marker 1 to Marker 2; a distance of 4.6 km on a bearing of 145° .

Stage 3 is from Marker 2 back to the Start.

The distance travelled on Stage 3, in km, is closest to

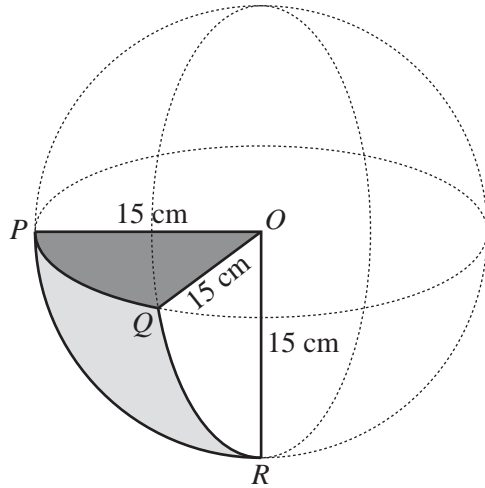
- A. 4.9
- B. 5.3
- C. 5.8
- D. 6.0
- E. 7.7

Question 9

The solid $OPQR$, as shown below, is one-eighth of a sphere of radius 15 cm.

The point O is the centre of the sphere and the points P , Q and R are on the surface of the sphere.

$$\angle POQ = \angle QOR = \angle ROP = 90^\circ$$

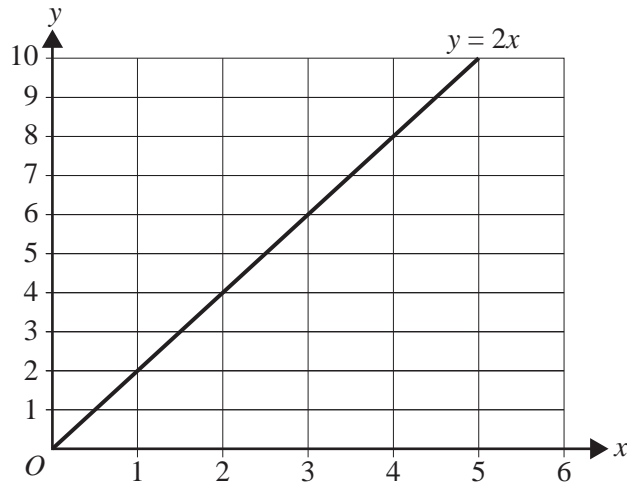


The total surface area of the solid $OPQR$, in cm^2 , is closest to

- A. 619
- B. 648
- C. 706
- D. 884
- E. 1767

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The line $y = 2x$ is drawn on the graph above.

Which one of the following points satisfies the inequality: $y > 2x$

- A. (1, 1)
- B. (2, 5)
- C. (3, 1)
- D. (4, 7)
- E. (5, 10)

Question 2

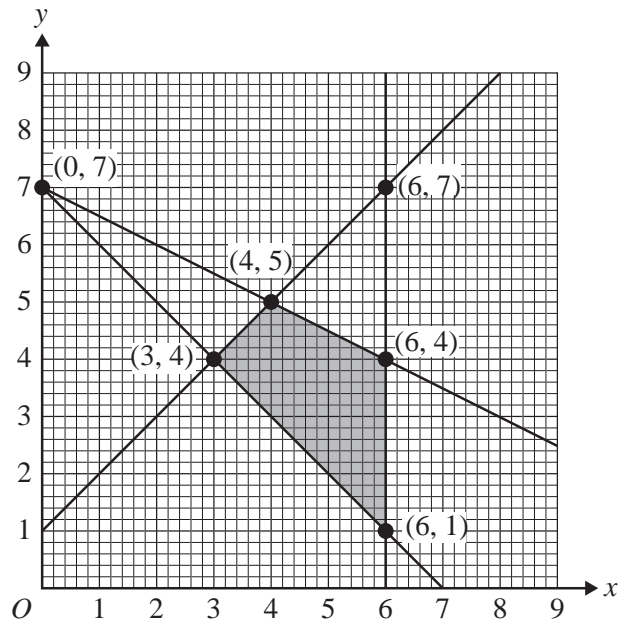
At a convenience store, one doughnut costs \$2.40 and one drink costs \$3.00.

A customer purchased five doughnuts and a number of drinks at a total cost of \$24.00.

The number of drinks purchased was

- A. 4
- B. 5
- C. 6
- D. 9
- E. 10

Question 3

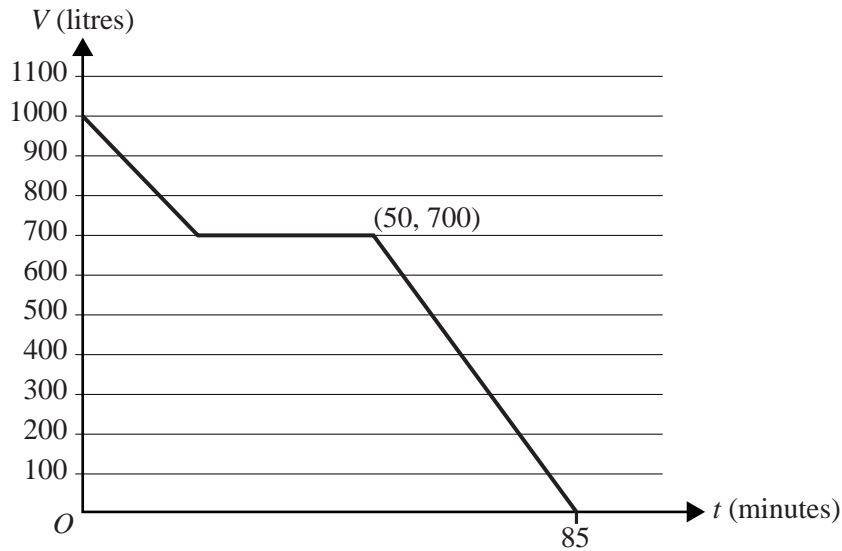


The shaded area in the graph above represents the feasible region for a linear programming problem. The minimum value of the objective $P = 2x - y$ for this feasible region is

- A. -7
- B. 0
- C. 1
- D. 2
- E. 3

Use the following information to answer Questions 4 and 5.

Question 4



The graph above shows the volume of water, V litres, in a tank at time t minutes.

The equation of this line between $t = 50$ and $t = 85$ minutes is

- A. $V = 1700 - 20t$
- B. $V = 700 - 20t$
- C. $V = 20t + 1700$
- D. $V = 20t + 700$
- E. $V = 35t - 700$

Question 5

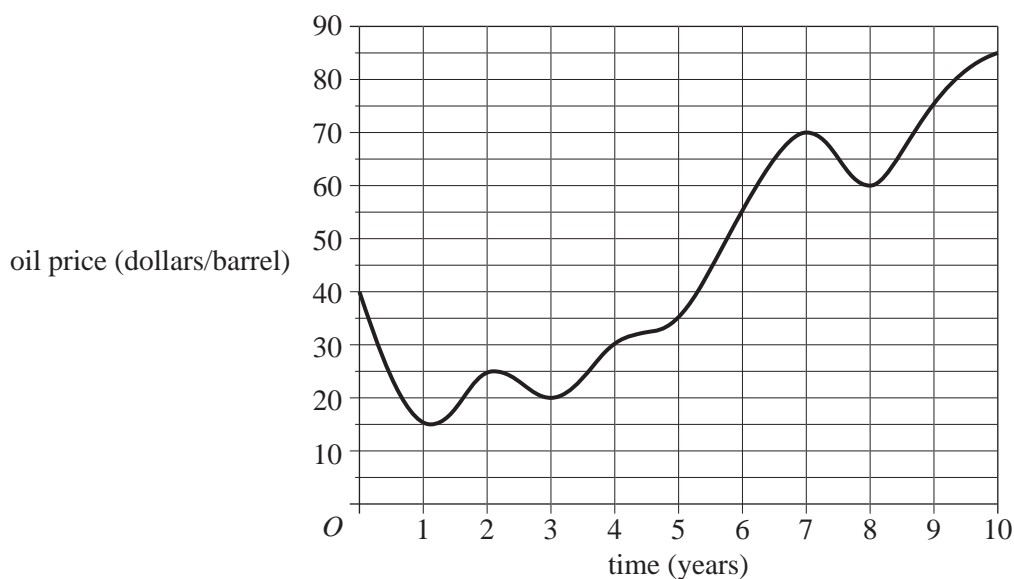
During the 85 minutes that it took to empty the tank, the volume of water in the tank first decreased at the rate of 15 litres per minute and then did not change for a period of time.

The period of time, in minutes, for which the volume of water in the tank did not change is

- A. 15
- B. 20
- C. 30
- D. 50
- E. 85

Question 6

The oil price (dollars/barrel) over a ten-year period is shown on the graph below.

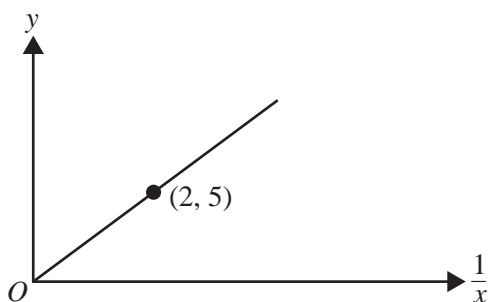


Which one of the following statements is **true**?

- A. The highest oil price over the ten-year period is \$70.
- B. The oil price decreased for exactly two of the ten years.
- C. The oil price changed most rapidly during the sixth year.
- D. On average, the oil price changed by \$4.50 per year over the ten years.
- E. The difference between the highest and lowest oil price during this ten-year period is \$45.

Question 7

A graph of y versus $\frac{1}{x}$ is shown below.



The rule connecting x and y is

- A. $y = \frac{5}{2x}$
- B. $y = \frac{5}{2}x$
- C. $y = \frac{2}{5x}$
- D. $y = \frac{2}{5}x$
- E. $y = \frac{10}{x}$

Question 8

Daisy's bread shop makes white and brown bread subject to the following constraints.

- No more than 240 loaves of bread can be made each day.
- At least five loaves of white bread will be made for every loaf of brown bread that is made.

Let w be the number of loaves of white bread that are made each day.

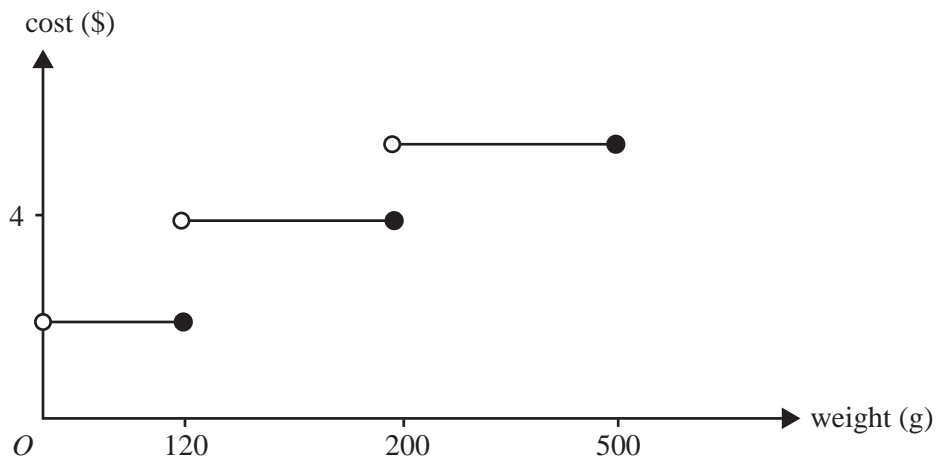
Let b be the number of loaves of brown bread that are made each day.

A pair of inequalities that could be written to represent these constraints is

- A. $w + b \leq 240$ and $w \geq 5b$
- B. $w + b \leq 240$ and $w \leq 5b$
- C. $w + b < 240$ and $w > \frac{b}{5}$
- D. $w + b < 240$ and $w < 5b$
- E. $w + b \leq 240$ and $w \leq \frac{b}{5}$

Question 9

The cost of posting a parcel that weighs 500 grams or less depends on its weight, as shown on the graph below.



The total cost of posting three separate parcels weighing respectively 100 grams, 150 grams and 210 grams is \$11.

The total cost of posting three separate parcels weighing respectively 60 grams, 110 grams and 350 grams is \$8.

The total cost of posting two separate parcels weighing respectively 170 grams and 500 grams is

- A. \$5
- B. \$7
- C. \$8
- D. \$10
- E. \$12

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The selling price of a large tin of paint is \$215.

After a 25% discount, the selling price of the tin of paint will become

- A. \$43.00
- B. \$53.75
- C. \$161.25
- D. \$190.00
- E. \$195.00

Question 2

\$3000 is invested at a simple interest rate of 6.5% per annum.

The total interest earned in three years is

- A. \$195.00
- B. \$580.50
- C. \$585.00
- D. \$3623.85
- E. \$3585.00

Question 3

The transaction details for a savings account for the month of July 2012 are shown below.

Date	Transaction	Debit (\$)	Credit (\$)	Balance (\$)
1 July	Opening balance			5201.82
5 July	Withdrawal	100.00		5101.82
20 July	Deposit		500.00	5601.82
23 July	Withdrawal	300.00		5301.82
31 July	Interest for the month of July		21.99	5323.81
31 July	Closing balance			5323.81

Interest is calculated and paid monthly on the minimum monthly balance.

The annual rate of interest paid on this account is closest to

- A. 3.5%
- B. 4.3%
- C. 4.7%
- D. 4.9%
- E. 5.2%

Question 4

Mei's starting salary is \$65 000 per annum.

After the first year her salary will increase by 2.8%.

After the second year her salary will increase by a further 3.5%.

After this second increase, her salary will be closest to

- A. \$66 820
- B. \$68 690
- C. \$69 030
- D. \$69 160
- E. \$69 630

Question 5

A second-hand car is purchased for \$9000.

A deposit of \$2500 is paid.

Interest is calculated at the rate of 14.95% per annum on the reducing monthly balance.

The balance and interest will be repaid over two years with equal monthly payments.

The monthly payment is closest to

- A. \$315
- B. \$415
- C. \$436
- D. \$575
- E. \$587

Question 6

The commission charged by a real estate agent for selling a property is calculated as shown in the table below.

Sale price	Commission
\$0 to \$100 000	3.5% of the sale price of the property
\$100 001 to \$250 000	\$3 500 plus 2% of the portion of the sale price greater than \$100 000
greater than \$250 000	\$6 500 plus 1.6% of the portion of the sale price greater than \$250 000

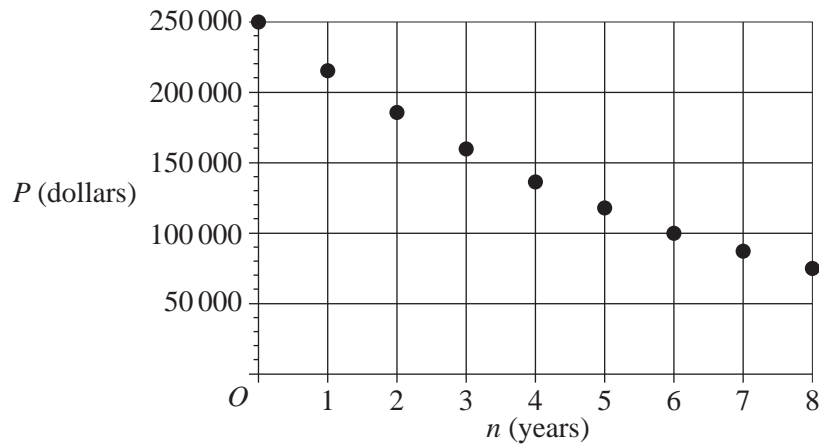
The sale price of a property is \$380 000.

The commission charged for this sale is

- A. \$6080
- B. \$7600
- C. \$8580
- D. \$9100
- E. \$10 980

Question 7

The following graph shows the decreasing value of an asset over eight years.



Let P be the value of the asset, in dollars, after n years.

A rule for evaluating P could be

- A. $P = 250\,000 \times (1 + 0.14)^n$
- B. $P = 250\,000 \times 1.14 \times n$
- C. $P = 250\,000 \times (0.14)^n$
- D. $P = 250\,000 \times (1 - 0.14)^n$
- E. $P = 250\,000 \times (1 - 0.14) \times n$

Question 8

\$15 000 is invested for 12 months.

For the first six months the interest rate is 6.1% per annum compounding monthly.

After six months the interest rate increases to 6.25% per annum compounding monthly.

The total interest earned by this investment over 12 months is closest to

- A. \$926
- B. \$935
- C. \$941
- D. \$953
- E. \$965

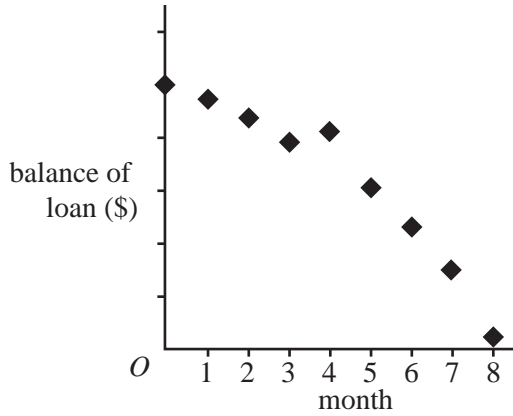
Question 9

Peter took out a reducing balance loan where interest was calculated monthly. He planned to repay this loan fully, with eight equal monthly payments of \$260.

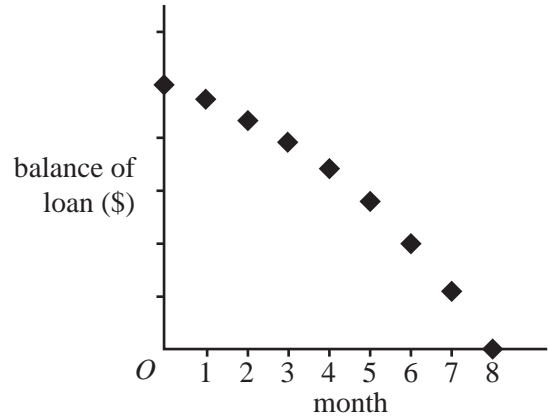
Peter missed the fourth payment, but made a **double** payment of \$520 in the fifth month. He then continued to make payments of \$260 for the remaining three months.

Which graph could show the balance of the loan each month over the eight-month period?

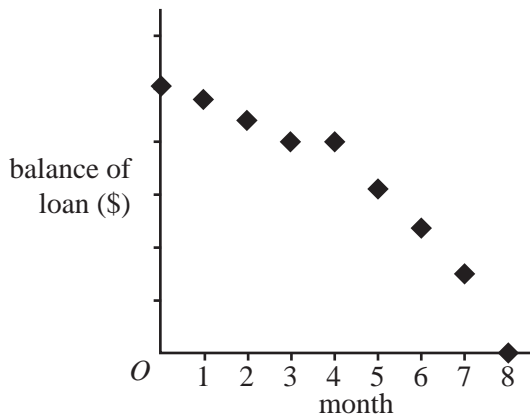
A.



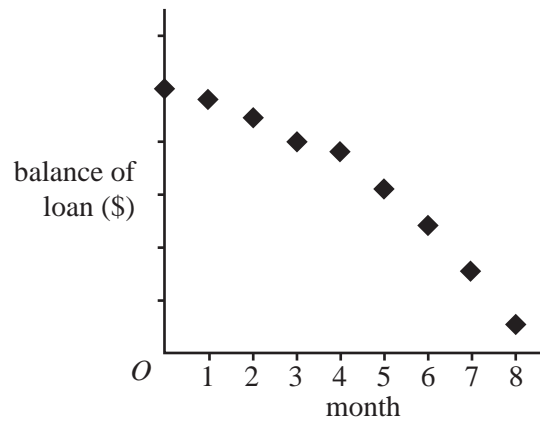
B.



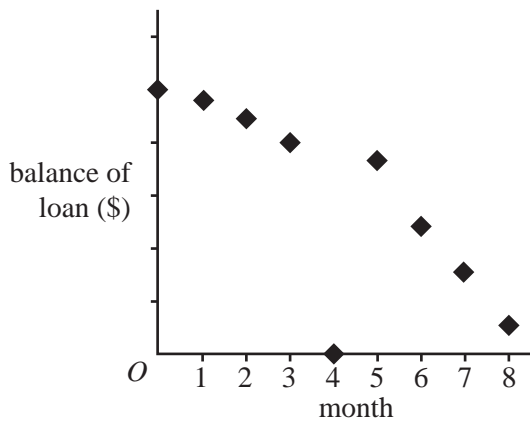
C.



D.



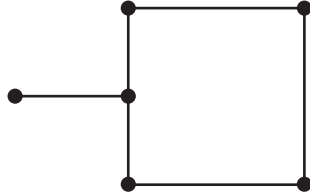
E.



Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

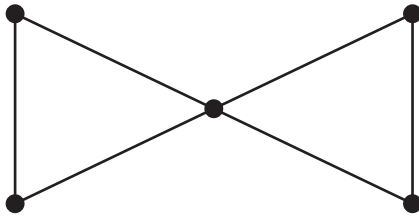
Question 1



The sum of the degrees of all the vertices in the graph above is

- A. 6
- B. 8
- C. 9
- D. 11
- E. 12

Question 2

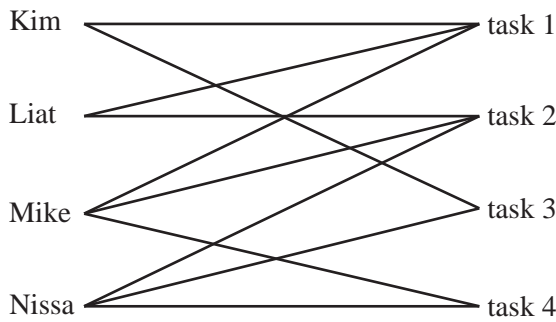


The number of Hamiltonian circuits involving all five vertices in the graph above is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 3

The bipartite graph below shows the tasks that each of four people is able to undertake.



All tasks must be allocated and each person can be allocated one task only.

A valid task allocation is

A.

Kim	task 1
Liat	task 2
Mike	task 3
Nissa	task 4

B.

Kim	task 3
Liat	task 1
Mike	task 2
Nissa	task 3

C.

Kim	task 3
Liat	task 2
Mike	task 1
Nissa	task 4

D.

Kim	task 1
Liat	task 3
Mike	task 4
Nissa	task 2

E.

Kim	task 2
Liat	task 1
Mike	task 4
Nissa	task 3

Question 4

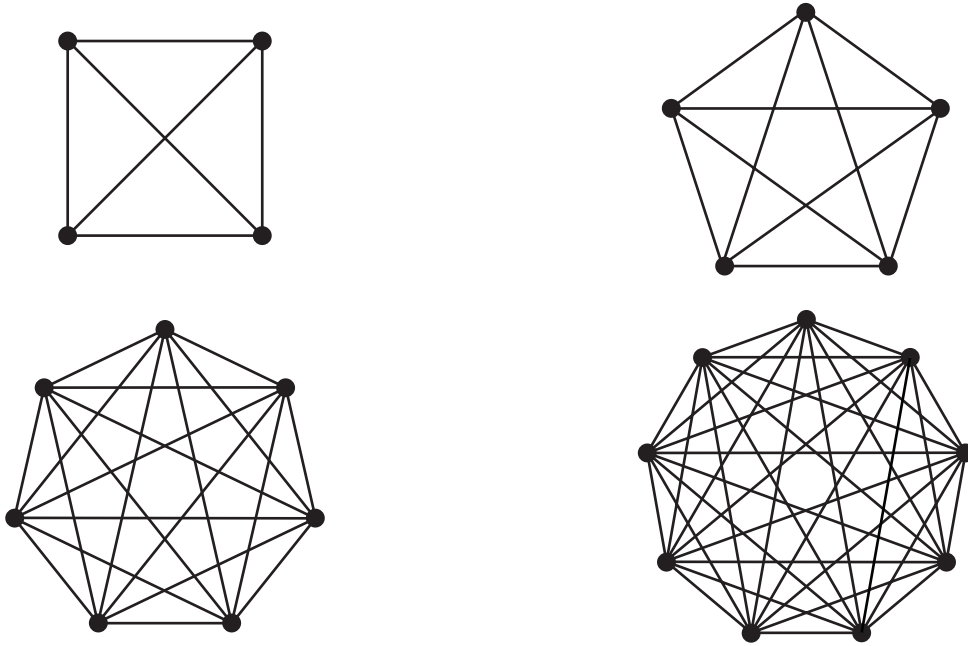
$$\begin{matrix}
 & P & Q & R & S \\
 P & \begin{bmatrix} 0 & 0 & 2 & 1 \end{bmatrix} \\
 Q & \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix} \\
 R & \begin{bmatrix} 2 & 1 & 0 & 1 \end{bmatrix} \\
 S & \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}
 \end{matrix}$$

The adjacency matrix above represents a planar graph with four vertices.

The number of faces (regions) on the planar graph is

- A.** 1
- B.** 2
- C.** 3
- D.** 4
- E.** 5

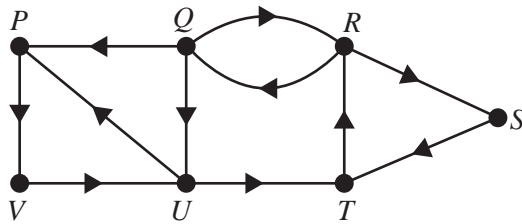
Question 5



How many of the four complete graphs above will have an Euler circuit?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 6



In the digraph above, all vertices are reachable from every other vertex.

All vertices would still be reachable from every other vertex if we remove the edge in the direction from

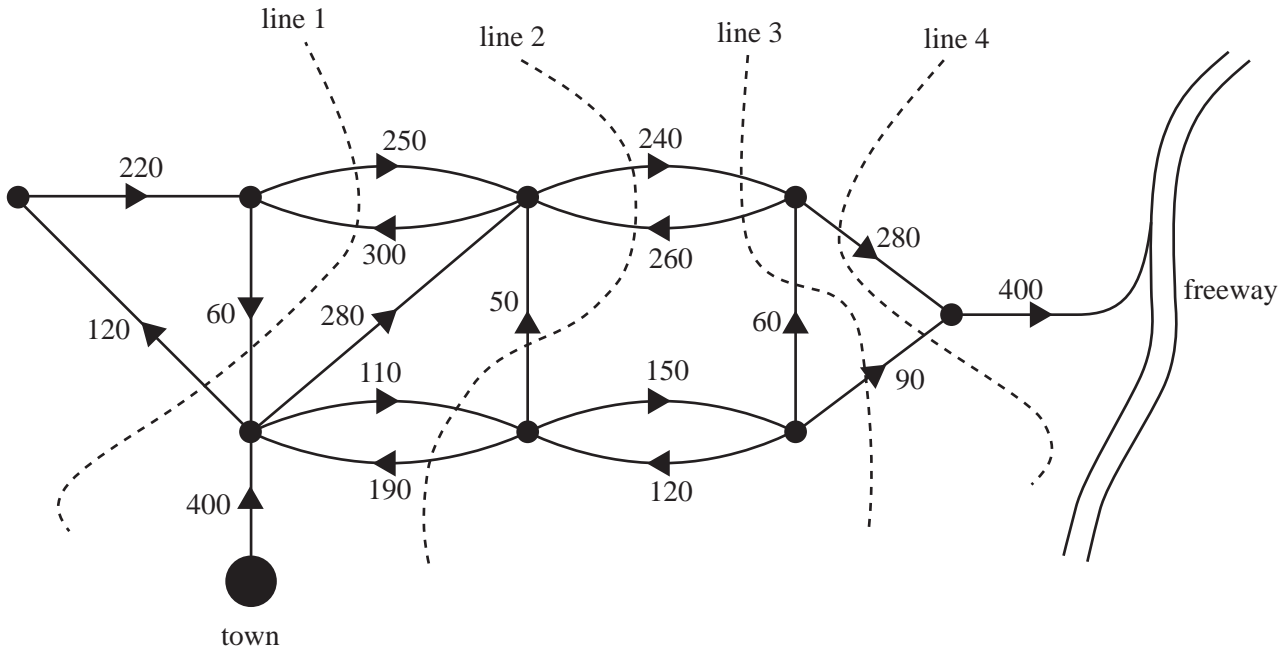
- A. Q to U
- B. R to S
- C. S to T
- D. T to R
- E. V to U

Question 7

Vehicles from a town can drive onto a freeway along a network of one-way and two-way roads, as shown in the network diagram below.

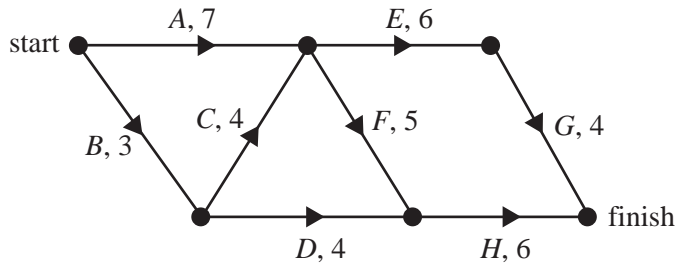
The numbers indicate the maximum number of vehicles per hour that can travel along each road in this network. The arrows represent the permitted direction of travel.

One of the four dotted lines shown on the diagram is the minimum cut for this network.



The maximum number of vehicles per hour that can travel through this network from the town onto the freeway is

- A. 310
- B. 330
- C. 350
- D. 370
- E. 390

Question 8

Eight activities, A , B , C , D , E , F , G and H , must be completed for a project.

The graph above shows these activities and their usual duration in hours.

The duration of each activity can be reduced by one hour.

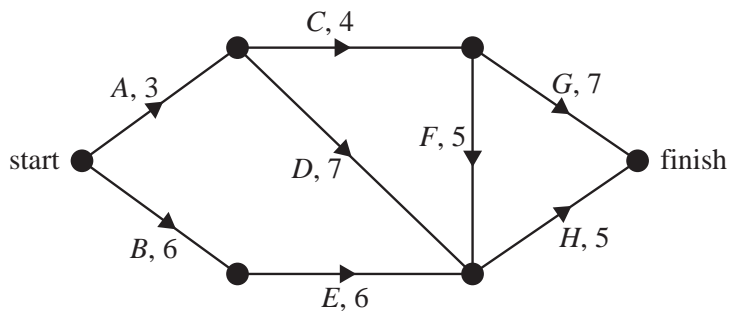
To complete this project in 16 hours, the minimum number of activities that must be reduced by one hour each is

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Question 9

John, Ken and Lisa must work together to complete eight activities, A , B , C , D , E , F , G and H , in minimum time.

The directed network below shows the activities, their completion times in days, and the order in which they must be completed.



Several activities need special skills. Each of these activities may be completed only by a specified person.

- Activities A and F may only be completed by John.
- Activities B and C may only be completed by Ken.
- Activities D and E may only be completed by Lisa.
- Activities G and H may be completed by any one of John, Ken or Lisa.

With these conditions, the minimum number of days required to complete these eight activities is

- A. 14
- B. 17
- C. 20
- D. 21
- E. 24

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

$$2 \times \begin{bmatrix} 2 & 8 \\ 4 & -1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ 4 & 2 \\ 2 & 3 \end{bmatrix} \text{ equals}$$

A. $\begin{bmatrix} 1 & 1 \\ 0 & -3 \\ 4 & 2 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 2 \\ 0 & -6 \\ 2 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 9 \\ 12 & 0 \\ 8 & 13 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 9 \\ 4 & -4 \\ 4 & 7 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 1 \\ 0 & -3 \\ 1 & 2 \end{bmatrix}$

Question 2

If $A = \begin{bmatrix} 8 & 1 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 12 \\ 6 & 0 \end{bmatrix}$, then the matrix $AB = \begin{bmatrix} 30 & 96 \\ 24 & 48 \end{bmatrix}$.

The element '24' in the matrix AB is correctly obtained by calculating

- A. $4 \times 6 + 2 \times 0$
- B. $4 \times 3 + 2 \times 6$
- C. $3 \times 4 + 12 \times 1$
- D. $4 \times 2 + 8 \times 2$
- E. $8 \times 3 + 1 \times 0$

Question 3

$$x + z = 6$$

$$2y + z = 8$$

$$2x + y + 2z = 15$$

The **solution** of the simultaneous equations above is given by

A.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & -1 & 2 \\ -2 & 0 & 1 \\ 4 & 1 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 15 \end{bmatrix}$$

B.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 15 \end{bmatrix}$$

C.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ -1 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 15 \end{bmatrix}$$

D.

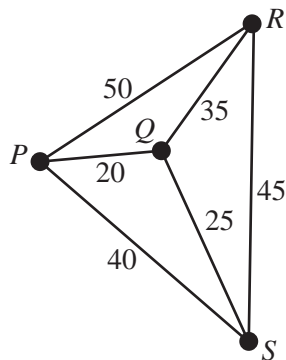
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 15 \end{bmatrix}$$

E.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 15 \end{bmatrix}$$

Question 4

The diagram below shows the tracks directly linking four camping sites P , Q , R and S in a national park. The shortest time that it takes to walk between the camping sites (in minutes), along each of these tracks, is also shown.



A matrix that could be used to present the same information is

A.

$$\begin{array}{cccc} P & Q & R & S \\ \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} P \\ Q \\ R \\ S \end{array} \end{array}$$

B.

$$\begin{array}{cccc} P & Q & R & S \\ \left[\begin{array}{cccc} 0 & 20 & 40 & 50 \\ 20 & 0 & 45 & 25 \\ 40 & 45 & 0 & 35 \\ 50 & 25 & 35 & 0 \end{array} \right] \begin{array}{l} P \\ Q \\ R \\ S \end{array} \end{array}$$

C.

$$\begin{array}{cccc} P & Q & R & S \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 20 & 1 & 0 & 0 \\ 50 & 35 & 1 & 0 \\ 40 & 25 & 45 & 1 \end{array} \right] \begin{array}{l} P \\ Q \\ R \\ S \end{array} \end{array}$$

D.

$$\begin{array}{cccc} P & Q & R & S \\ \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{array} \right] \begin{array}{l} P \\ Q \\ R \\ S \end{array} \end{array}$$

E.

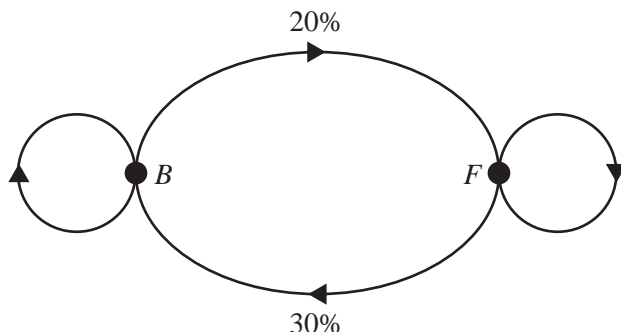
$$\begin{array}{cccc} P & Q & R & S \\ \left[\begin{array}{cccc} 0 & 20 & 50 & 40 \\ 20 & 0 & 35 & 25 \\ 50 & 35 & 0 & 45 \\ 40 & 25 & 45 & 0 \end{array} \right] \begin{array}{l} P \\ Q \\ R \\ S \end{array} \end{array}$$

Question 5

There are two fast-food shops in a country town: Big Burgers (B) and Fast Fries (F).

Every week, each family in the town will purchase takeaway food from one of these shops.

The transition diagram below shows the way families in the town change their preferences for fast food from one week to the next.



A transition matrix that provides the same information as the transition diagram is

A.

$$\begin{array}{c} \text{from} \\ B \quad F \\ \left[\begin{array}{cc} 0 & 0.3 \\ 0.2 & 0 \end{array} \right] \begin{array}{l} B \\ F \end{array} \text{ to} \end{array}$$

B.

$$\begin{array}{c} \text{from} \\ B \quad F \\ \left[\begin{array}{cc} 1.2 & -0.3 \\ -0.2 & 1.3 \end{array} \right] \begin{array}{l} B \\ F \end{array} \text{ to} \end{array}$$

C.

$$\begin{array}{c} \text{from} \\ B \quad F \\ \left[\begin{array}{cc} 0.7 & 0.3 \\ 0.2 & 0.8 \end{array} \right] \begin{array}{l} B \\ F \end{array} \text{ to} \end{array}$$

D.

$$\begin{array}{c} \text{from} \\ B \quad F \\ \left[\begin{array}{cc} 0.8 & 0.3 \\ 0.2 & 0.7 \end{array} \right] \begin{array}{l} B \\ F \end{array} \text{ to} \end{array}$$

E.

$$\begin{array}{c} \text{from} \\ B \quad F \\ \left[\begin{array}{cc} 0.7 & 0.2 \\ 0.3 & 0.8 \end{array} \right] \begin{array}{l} B \\ F \end{array} \text{ to} \end{array}$$

Question 6

The table below shows the number of classes and the number of students in each class at each year level in a secondary school.

	Year level			
	9	10	11	12
Number of classes	7	5	6	4
Students per class	22	20	18	24

Let

$$F = [1 \ 1 \ 1 \ 1], \quad G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad M = [7 \ 5 \ 6 \ 4], \quad N = \begin{bmatrix} 7 \\ 5 \\ 6 \\ 4 \end{bmatrix}, \quad P = \begin{bmatrix} 22 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 24 \end{bmatrix}$$

A matrix product that displays the total number of students in Years 9–12 at this school is

- A. $M \times P \times F$
- B. $P \times G \times M$
- C. $F \times P \times N$
- D. $P \times N \times F$
- E. $F \times N \times P$

Question 7

A store has three outlets, *A*, *B* and *C*. These outlets sell dresses, jackets and skirts made by the fashion house Ocki.

The table below lists the number of Ocki dresses, jackets and skirts that are currently held at each outlet.

	Size 10	Size 12	Size 14	Size 16
Outlet A	2 dresses	3 jackets	1 skirt	4 jackets
Outlet B	1 skirt	1 jacket	3 jackets	1 dress
Outlet C	2 skirts	2 dresses	2 dresses	1 jacket

A matrix that shows the total number of Ocki dresses (*D*), jackets (*J*) and skirts (*S*) in each size held at the three outlets is given by

A.

$$\begin{array}{l}
 D \quad J \quad S \\
 \text{Size 10} \\
 \text{Size 12} \\
 \text{Size 14} \\
 \text{Size 16}
 \end{array}
 \begin{bmatrix}
 2 & 1 & 2 \\
 3 & 1 & 2 \\
 1 & 3 & 2 \\
 4 & 1 & 1
 \end{bmatrix}$$

B.

$$\begin{array}{l}
 D \quad J \quad S \\
 \text{Size 10} \\
 \text{Size 12} \\
 \text{Size 14} \\
 \text{Size 16}
 \end{array}
 \begin{bmatrix}
 2 & 0 & 3 \\
 2 & 4 & 0 \\
 2 & 3 & 1 \\
 1 & 5 & 0
 \end{bmatrix}$$

C.

$$\begin{array}{l}
 D \quad J \quad S \\
 \text{Size 10} \\
 \text{Size 12} \\
 \text{Size 14} \\
 \text{Size 16}
 \end{array}
 \begin{bmatrix}
 2 & 3 & 1 \\
 4 & 1 & 1 \\
 3 & 1 & 2 \\
 2 & 2 & 1
 \end{bmatrix}$$

D.

$$\begin{array}{l}
 D \quad J \quad S \\
 \text{Size 10} \\
 \text{Size 12} \\
 \text{Size 14} \\
 \text{Size 16}
 \end{array}
 \begin{bmatrix}
 7 & 0 & 0 \\
 0 & 12 & 0 \\
 0 & 0 & 4 \\
 0 & 0 & 0
 \end{bmatrix}$$

E.

$$\begin{array}{l}
 D \quad J \quad S \\
 \text{Size 10} \\
 \text{Size 12} \\
 \text{Size 14} \\
 \text{Size 16}
 \end{array}
 \begin{bmatrix}
 1 & 0 & 2 \\
 2 & 1 & 0 \\
 1 & 1 & 1 \\
 1 & 2 & 0
 \end{bmatrix}$$

Question 8

There are 30 children in a Year 6 class. Each week every child participates in one of three activities: cycling (*C*), orienteering (*O*) or swimming (*S*).

The activities that the children select each week change according to the transition matrix below.

$$T = \begin{array}{ccc} & \begin{array}{ccc} \textit{this week} \\ C & O & S \end{array} & \\ \begin{array}{c} C \\ O \\ S \end{array} & \begin{bmatrix} 0.5 & 0.3 & 0.3 \\ 0.1 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} & \begin{array}{c} C \\ O \\ S \end{array} \end{array} \begin{array}{c} \\ \\ \textit{next week} \end{array}$$

From the transition matrix it can be concluded that

- A. in the first week of the program, ten children do cycling, ten children do orienteering and ten children do swimming.
- B. at least 50% of the children do not change their activities from the first week to the second week.
- C. in the long term, all of the children will choose the same activity.
- D. orienteering is the most popular activity in the first week.
- E. 50% of the children will do swimming each week.

Question 9

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} a \\ 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ b \end{bmatrix}$$

Which set of equations below could be used to determine the values of a and b that are shown in the matrix equation above?

- A. $a - b = 2$
 $a + b = 0$
- B. $a + b = -2$
 $a - b = 0$
- C. $a + b = 2$
 $a - b = 0$
- D. $a - b = 8$
 $a + b = 2$
- E. $a - b = 8$
 $a + b = -2$

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics formulas

Core: Data analysis

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \quad \text{where } b = r \frac{s_y}{s_x} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

residual value:
$$\text{residual value} = \text{actual value} - \text{predicted value}$$

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

geometric series:
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, \quad |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2}bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s - a)(s - b)(s - c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3}\pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3}\pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
$$\text{area of base} \times \text{height}$$

volume of a pyramid:
$$\frac{1}{3} \text{area of base} \times \text{height}$$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$