2012 Further Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The majority of students seemed to be well prepared for Further Mathematics examination 1 in 2012.

SPECIFIC INFORMATION

Section A
Core – Data analysis

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer
1	1	0	0	0	99	0
2	5	68	6	2	19	0
3	3	2	6	1	88	0
4	7	4	16	68	5	1
5	3	5	31	59	2	0
6	3	84	7	2	4	0
7	4	11	8	70	7	0
8	73	6	6	3	11	0
9	12	42	4	34	8	0
10	9	10	19	51	11	0
11	11	9	60	12	8	0
12	62	7	9	6	16	0
13	3	10	39	6	41	0

The questions in this section of the paper were generally well answered, with the exceptions of Questions 9 and 13.

In Question 9, students were asked to apply five-median smoothing to a time series plot. Of those students who failed to answer the question correctly, a significant proportion chose the answer that most closely matched five-mean smoothing, suggesting a lack of care when reading the question.

In Question 13, students were asked to use a trend line relating deseasonalised quarterly sales to quarter number in order to predict the actual sales in quarter 4.

A possible solution strategy is as follows.

Use the trend line to predict the deseasonalised sales in quarter 4.

deseasonalised sales in quarter 4 = 256 000 + 15 6000 × 4 = 318 400

To find the actual sales from the deseasonalised sales, the seasonal index for quarter 4 is needed.

seasonal index $_{\text{quarter 4}}$ = 4 – (1.2 + 0.7 + 0.8) = 1.3

Thus

actual sales = *deseasonalised sales* × seasonal index = 318 400 × 1.3 = 413 920

Section B

Module 1 – Number patterns

Question	% A	% B	% C	% D	% E	% No Answer
1	7	5	4	82	1	0
2	8	3	19	3	67	0
3	85	9	1	4	1	0
4	10	11	10	65	3	1
5	33	9	48	4	6	1
6	9	10	8	9	64	1
7	12	13	15	40	19	1
8	18	50	19	7	6	1
9	50	17	20	8	4	1

The questions in Module 1: Number patterns were generally well answered, with the exception of Question 7.

In Question 7, students were asked to find the greatest speed reached by a dragster that is travelling at a speed of 100km/h and increasing its speed each second according to a given pattern.

One possible solution strategy is as follows.

The increase in speed each second (50 km/h, 30 km/h, 18 km/h…) forms an infinite geometric sequence with first term a = 50 km/h and common ratio $r = \frac{30}{50} = 0.6$.

The sum of this infinite sequence is $S = \frac{50}{1 - 0.6} = 125$, so the maximum increase in speed cannot exceed 125 km/h.

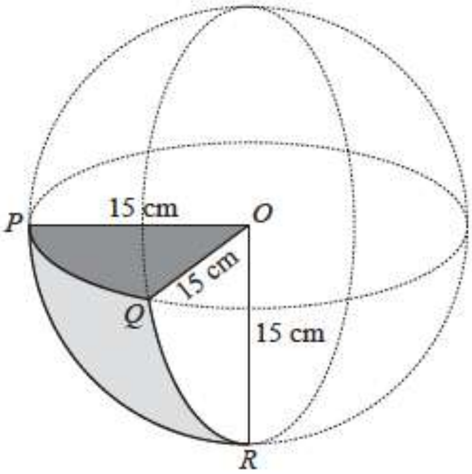
Thus, the greatest speed the dragster could reach is 100 + 125 = 225 km/h.

Module 2 – Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	3	18	67	11	1	0
2	4	5	8	73	10	0
3	10	60	14	10	5	1
4	8	62	12	9	8	1
5	11	20	11	44	13	1
6	8	3	23	47	19	0
7	11	7	61	10	10	0
8	10	11	58	10	10	1
9	13	12	33	31	10	1

The questions in Module 2: Geometry and trigonometry were generally well answered, with the exception of Question 9.

Question 9 was challenging for many students. In this question, students were asked to determine the total surface area of a solid that constituted one-eighth of the volume of a sphere, as shown below.



One possible approach is as follows.

The solid is bounded by the curved surface *PQR* and three sectors of a circle of equal area *QOR*, *POQ* and *POR* (hidden). Thus

total surface area required = area of curved surface *PQR* + 3 × area of sector *QOR*

$= \frac{1}{8} \times$ the surface area of the sphere of radius 15 cm + $3 \times \frac{1}{4} \times$ the area of the circle of radius 15 cm

$= \frac{1}{8} \times 4\pi \times 15^2 + 3 \times \frac{1}{4} \times \pi \times 15^2$

$= 883.57 \ldots \text{ cm}^2$

Module 3 – Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	3	63	10	2	22	0
2	96	1	1	1	0	0
3	6	3	11	73	7	0
4	49	26	4	6	14	0
5	6	20	64	7	2	1
6	1	7	29	49	14	0
7	40	25	9	10	15	1
8	55	14	6	3	21	1
9	4	9	11	66	10	1

Module 3: Graphs and relations was generally well done, with the exception of Question 7.

In Question 7, students were asked to find the relationship between two variables, *y* and *x*, given that a graph of *y* plotted against $\frac{1}{x}$ was a straight line passing through the point (2, 5). The correct answer was $y = \frac{5}{2x}$. A significant number of students chose the incorrect answer $y = \frac{5x}{2}$ by ignoring the fact that the graph was a plot of *y* against $\frac{1}{x}$, not *x*.

Module 4 – Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	1	16	81	2	1	0
2	3	2	90	2	2	0
3	12	31	6	11	39	2
4	4	5	4	83	3	0
5	65	12	14	5	4	1
6	10	7	70	6	7	1
7	12	8	8	63	8	1
8	16	16	16	42	8	1
9	20	4	42	6	27	1

Although most of the questions in Module 4: Business and related mathematics were generally well answered, Question 3 was not done well. Question 9 was very poorly answered.

In Question 3, students were given the transaction details for a savings account for one month. For this account, interest was paid monthly on the minimum monthly balance. Using this information, students were asked to determine the annual rate of interest paid on this account.

One possible solution strategy is as follows.

Reading from the transaction details for the account,
interest paid for the month: $21.99
minimum monthly balance: $5101.82

If r is the annual rate of interest, then $21.99 = 5101.82 \times \frac{r}{12} \times \frac{1}{100}$ or $r = 5.172\ldots$

In Question 9, students were asked to identify the graph that correctly represented the decreasing monthly balance of a loan when the fourth monthly payment is missed but made up with a double payment the following month. The key to answering this question was to recognise that the balance of the loan increases in the fourth month because, while no payment is made, interest is still charged and added to the amount owed.

Module 5 – Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	15	1	1	8	76	0
2	60	5	8	3	25	0
3	2	8	86	2	3	0
4	1	6	10	63	20	0
5	9	7	11	67	6	0
6	74	7	9	6	4	0
7	28	10	23	29	9	0
8	12	43	30	8	6	0
9	11	45	14	18	12	1

While the first six questions in Module 5: Networks and decision mathematics were well answered, Questions 7, 8 and 9 were challenging for many students.

Question 7 required students to determine the maximum possible flow in a network by identifying the minimum cut. The key to answering this question was to recognise that to apply the 'minimum cut-maximum flow' theorem, the cut must separate the 'source' of traffic (the town) from the 'sink' (the freeway). Of the cuts shown on the network, only those represented by lines 2, 3 and 4 separated the source from the sink and could potentially be used to determine the maximum flow. The solution then proceeds as follows.

Line 1 does not separate the source from the sink.
Flow across the cut represented by line 2 = 240 + 110 = 350
Flow across the cut represented by line 3 = 240 + 60 + 90 = 390
Flow across the cut represented by line 4 = 280 + 90 = 370

Thus, line 2 is the minimum cut, implying that the maximum flow of vehicles through the network from the town to the freeway is 350 per hour.

Questions 8 and 9 both involved critical path analysis.

In answering Question 8, it was necessary to realise that there are two paths, AFH and BCFH, of length 18 hours (the critical paths) and two paths, AEG and BCEG, of length 17 hours. Reducing activities A and B by one hour each reduces the lengths of these two paths to 17 and 16 hours respectively. Reducing either F or H by one hour will then further reduce the critical paths AFH and BCFH to 16 hours as required. Thus a minimum of three activities must be reduced by one hour each to reduce the project completion time to 16 hours. It was not sufficient to just reduce the length of paths AFH and BCFH to 16 hours, for example by reducing the durations of F and H by one hour each because it would still take 17 hours to complete the activities on the paths AEG and BCEG.

One strategy for answering Question 9 is to begin by determining the minimum time required to complete the project without any delays caused by restrictions on who can do particular activities.

Without restrictions, the critical paths are ACFH and BEH, giving a minimum completion time of 17 days.

With restrictions

- the length of time required to complete path ACFH is increased by three days because Ken must complete activity B before starting activity C, giving a revised completion time of 17 + 3 = 20 hours. Requiring John to complete task A does not change this
- the length of time required to complete path BEH is increased by four days because Lisa must complete activity D before starting activity E, giving a revised completion time of 17 + 4 = 21 hours. Requiring Ken to complete task B does not change this.

Thus the minimum completion time required for the project with the restrictions given is 21 hours.

Module 6 – Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	1	95	1	0
2	20	72	4	1	2	0
3	41	49	4	3	2	0
4	1	1	2	12	84	0
5	13	1	6	74	5	0
6	10	13	60	10	6	1
7	22	72	3	2	1	0
8	8	55	11	11	14	1
9	13	39	25	13	9	1

With the exception of Question 9, Module 6: Matrices was generally very well done.

In Question 9, students were asked to identify a set of equations that could be solved to determine the values of a and b in the matrix equation below.

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ b \end{bmatrix}$$

One possible solution strategy is as follows.

Simplify the equation by completing the matrix products on each side of the equation.

$$\begin{bmatrix} 3a + 12 \\ a + 6 \end{bmatrix} = \begin{bmatrix} 12 + 3b \\ 4 - b \end{bmatrix}$$

Thus

$3a + 12 = 12 + 3b$ or $a - b = 0$

and

$a + 6 = 4 - b$ or $a + b = -2$