

MINIMUM SPANNING TREES



Ex10 H
Notes

A A tree is \rightarrow connected graph

\rightarrow NO loops \rightarrow ~~x~~

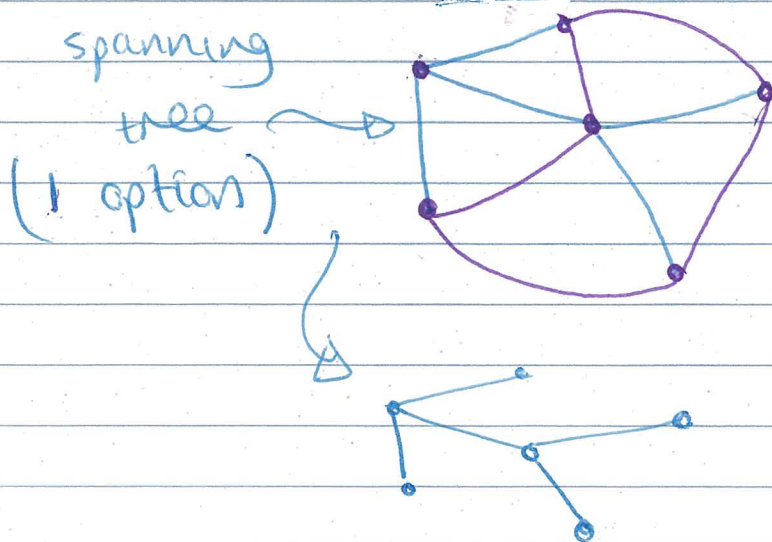
\rightarrow NO multiple edges \rightarrow ~~x~~

\rightarrow NO circuits \rightarrow ~~x~~

but it can be part of a larger graph.

For n vertices, the tree has $(n-1)$ edges

B A spanning tree - connects ALL vertices
(no loops, multiple edges or circuits)



* there are other options *

But how do I find a spanning tree??
hmmm.... excellent question!!

use the relationship between vertices and edges i.e. n and $n-1$

so in previous graph, there are 6 vertices
so I need $(6-1) = 5$ edges.

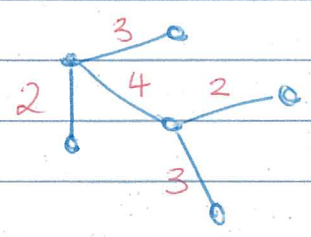
So then take out edges to get back to required number

* just make sure the conditions for a tree are still being met ok! :)

In Ex 10 A, we looked at "weighted graphs".
we can use this knowledge to work out the length of the spanning tree.

Simply add to numbers on the edges!

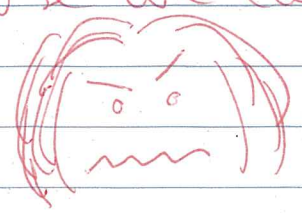
yay!



Length = $2 + 3 + 4 + 2 + 3$
 $= 14$ units

So we can compare lengths of different spanning trees to find the shortest one

Of course we can m/s Vincent....



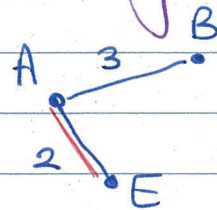
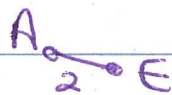
Prim's Algorithm

→ rules to follow for minimum spanning tree

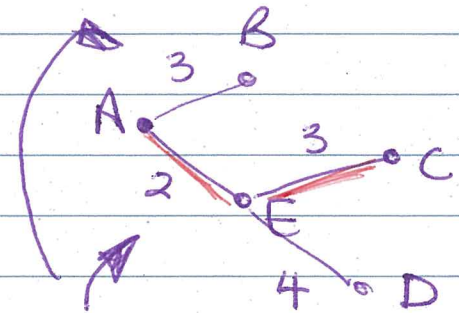
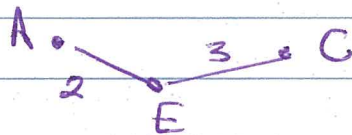
① Choose any vertex



② Follow edge with lowest weighting to next vertex



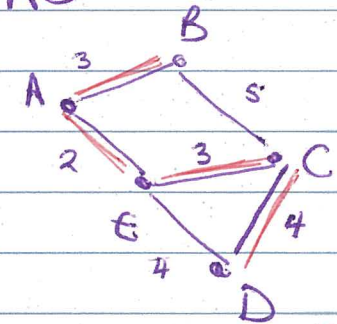
③ Choose the lowest edge coming off your 2 vertices



could choose either EC or AB

now choose lowest from A, E & C

④ Repeat until all vertices connected
→ choose AB, then CD



NOTE: You CANNOT choose an edge that forms a cycle

Each step adds another vertex to consider: ALWAYS choose the lowest number path

check 5 vertices
∴ 5-1 edges required

Finally, add up the weightings to find length:

$$\begin{aligned} \text{Length} &= 3 + 2 + 3 + 4 \\ &= 12 \text{ units} \end{aligned}$$

NOW TRY EX 10 H questions

