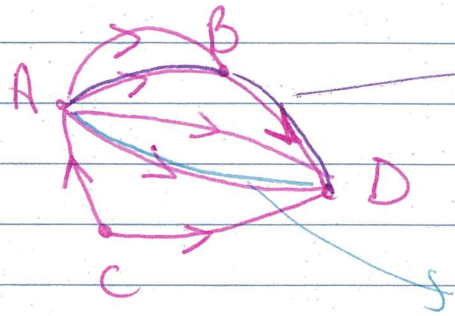
DIRECTED GRAPHS (or digraphs)

now we look at graphs (or networks) where the edges have a direction

* Think 1 way streets!!

Reachability: to do with analysing the various paths between vertices.



the indicated route from A to D is called two-stage

This route from A to D is one stage

A has 4 routes out → outdegree is 4
A has 1 route in → indegree is 1

Create adjacency matrix

- this shows all one-stage paths between vertices

[A]

* direction is important so there are 0 paths from B to A but 2 from A to B.

TO

$$
\text{FROM}\quad
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
A & 0 & 2 & 0 & 1 \\
B & 0 & 0 & 0 & 1 \\
C & 1 & 0 & 0 & 1 \\
D & 0 & 0 & 0 & 0
\end{array}
$$

sum row = out degree

sum column = indegree

matrix A is the adjacency matrix
A^2 will give you the 2-stage paths.

remember how to store in your CAS & square it.

If we square the matrix from before:

$$\begin{bmatrix} 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{array}{cc} & \text{TO} \\ \text{From} & \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

don't need labels anymore

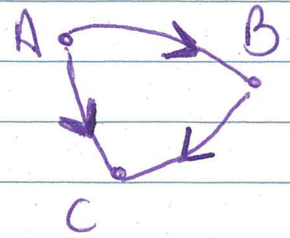
This gives the 2-stage paths
so from A to D there are 2 two-stage
from C to B there are 2 two-stage paths

etc.

This matches our digraph phew!

So $[A]^3$ would give 3-stage
$[A]^4$ would give 4-stage etc. etc...

Dominance in digraphs



In this graph,
A is dominant over B
and C

B is dominant over C

To determine dominant vertex in a network
use matrices ← OK !!

Work out the adjacency matrix [A]

then $A + A^2$ = resultant matrix ← this gives the sum of all 1 and 2 stage paths

then add up each row
↳ biggest number ⇒ dominant vertex

Lets look at the network on page 1

$$A + A^2 = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \leftarrow \text{sum is } 6 \\ \leftarrow \text{sum is } 1 \\ \leftarrow \text{sum is } 6 \\ \leftarrow \text{sum is } 0 \end{matrix}$$

This indicates that for this graph, vertices A and C are both equally dominant.

This can also be seen by inspection
→ look at how many one-stage paths
→ and look at the two-stage paths

Not always easy to tell just by looking so be careful

OK ?

Try questions from booklet
Ch15 Ex15A