

Solving Simultaneous Equations using Matrices

A key application for matrices is to use them for solving linear equations

To do this you need to be able to find the determinant and interpret it AND find the inverse of a matrix.

$$\text{determinant of } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{is } \begin{aligned} & axd - bxc \\ & = ad - bc \end{aligned}$$

← to find manually

or use CAS.

$$\text{inverse of } A = A^{-1}$$

$$= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

← to find manually

or use CAS

If det of matrix = 0 \Leftarrow is called singular
there is NO UNIQUE solution

This means could be

- infinite solutions (same line)
- no solution (parallel lines)

Example:
Use matrices to solve:

$$\begin{aligned}
 4x - 3y &= 10 \\
 y + 3x &= 1 \quad \leftarrow \text{rearrange this to} \\
 3x + y &= 1
 \end{aligned}$$

So equations are:

coefficient matrix is

$$\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}$$

(Arrows from the equations above point to the corresponding elements in the matrix: 4x to 4, -3y to -3, y to 1, and 3x to 3.)

So set up as:

$$\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

Then use CAS to solve

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

(A green circle around the inverse symbol has an arrow pointing to the text "means inverse".)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

solution is
 $x=1, y=-2$

*You may be asked to show how you know there is a solution

by hand the inverse is

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix}$$

$$\begin{aligned}
 \det &= 4 \times 1 - (-3 \times 3) \\
 &= 4 + 9 \\
 &= 13
 \end{aligned}$$

(The result 13 is circled in green.)

as $\det \neq 0$
there is
unique
solution