Solving Simultaneous Equations using Matrices

A key application for matrices is to use them for solving linear equations

To do this you need to be able to find the determinant and interpret it **AND** find the inverse of a matrix.

determinant of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

is $a \times d - b \times c$ ← to find manually
$= ad - bc$

or use CAS.

inverse of A $= A^{-1}$

$= \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ← to find manually

or use CAS

If det of matrix $= 0$ ⇐ is called singular
there is NO UNIQUE solution

This means could be

- infinite solutions (same line)
- no solution (parallel lines)

Example:
Use matrices to solve:

$4x - 3y = 10$
$y + 3x = 1$ ← rearrange this to

$3x + y = 1$

So equations are: $4x - 3y = 10$
$3x + 1y = 1$

coefficient matrix is $\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}$

So set up as: $\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$

Then use CAS to solve

means inverse

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ solution is $x = 1, y = -2$

*You may be asked to show how you know there is a solution

$\det = 4 \times 1 - (-3 \times 3)$
$= 4 + 9$
$= 13$

as $\det \neq 0$ there is unique solution

by hand the inverse is

$A^{-1} = \frac{1}{13}\begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix}$